SMALL SOLID BODY VOLUME AND DENSITY MEASUREMENT USING HYDROSTATIC WEINGHING METHOD

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Abstract: A simple hydrostatic weighing system was assembled. The pure water was used as reference material for density. The real mass of solid body was used instead of the measurement of apparent mass in the air. The results was good enough to distinguish density of different type of stainless steel.

Key words: hydrostatic weighing, solid volume, density.

1. INTRODUCTION

The hydrostatic weighing system for measurement of liquid density was implemented to provide traceability for calibration of digital density meter, volume standard and flow meters. It is operated manually and has flexible applicability to measure volume of solid pieces and density of liquids and solid body.

The pure water was used as reference of density to measure the volume of solid body. The density was determined as mass divided by volume.

The solid body can be used as standard volume to measure the liquid density using the same system.

2. METHOD

Instead of weighing the solid body in the air and in the liquid to determine apparent mass in two conditions, its real mass [1] was used. Using this method, it is necessary measure only the apparent mass [1] in the liquid to determined the liquid buoyancy force. If the liquid density is known the solid volume can be determined. If the solid volume is known the liquid density can be determined.

The mathematical model of measurement become simpler for using the real mass, that can be determined by mass laboratory with lower uncertainty.

The hydrostatic weighing system is compounded by: thermometer, balance, concrete structure, thermal bath, elevation apparatus, weighing device by bottom side of balance, stem, and others.

2.1. Determination of pure water density

The Tanaka equation [2] recommended by BIPM/CIPM was used to calculate the pure water density.

Equation of Tanaka

$$\rho_L(T_L) = a_5 \cdot \left[1 - \frac{(T_L + a_1)^2 \cdot (T_L + a_2)}{a_3 \cdot (T_L + a_4)}\right] + \delta\rho w \qquad (1)$$

$\boldsymbol{\rho_L}$ is the pure water density at temperature $\boldsymbol{T_L}$;
$\boldsymbol{T_L}$ is the water temperature [ºC];
$\boldsymbol{a_1}$ is -3,983 035 ºC;
$\boldsymbol{a_2}$ is 301,797 ºC;
$\boldsymbol{a_2}$ is 522 528,9 ºC;
$\boldsymbol{a_3}$ is 69,348 ºC;
$\boldsymbol{a_4}$ is 999,974 95 kg . m^{-3};
$\boldsymbol{\delta\rho_w}$ is the error due to water impurity (this term does not take part of original equation).

2.2. Mathematical model for the solid body volume and density measurement

The volume of solid body (volume standard) can be determined using the pure water as density reference material by equation (2).

$$V_r = \frac{M_R - M_L}{\rho_L} \cdot \left(1 + \alpha \cdot (T_r - T_L)\right) + \delta V + \delta V_S + \delta V_\sigma \qquad (2)$$

Where:

$\boldsymbol{V_r}$ is the measured volume standard at reference temperature "$\boldsymbol{T_r}$" [cm^3]
$\boldsymbol{M_R}$ is the real mass of volume standard [g]
$\boldsymbol{M_L}$ is the apparent mass on balance for volume standard immersed in liquid [g]
$\boldsymbol{\rho_L}$ is the liquid density during measurement of $\boldsymbol{M_L}$ at temperature "$\boldsymbol{T_L}$" [g cm^{-3}]

α is the volumetric coefficient of thermal expansion of volume standard [$^oC^{-1}$]

T_L is the liquid temperature during measurement of volume standard [^oC]

T_r is the temperature of reference of volume standard [^oC]

δV is the error due to repeatability/reproducibility [cm^3]

δV_S is the error due to volume difference between volume support and counterweight [cm^3]

δV_σ is the error due to variation of liquid surface tension force on the wire [cm^3]

The density of volume standard at reference temperature is calculated as following equation:

$$\rho_o = \frac{M_r}{V_R} \qquad (3)$$

Where:

ρ_o is the density of solid volume [g cm^{-3}]

2.3. Procedures for the solid body volume and density measurement

The measurement procedure is the following:

a) the liquid in the thermostatic bath must have their temperature in equilibrium and stable;

b) the weighing device with the counterweight must be immersed in the liquid (the counterweight has to have the same mass and volume of the support of the volume standard);

c) the volume standard, with its support, should be immersed in the liquid with its temperature stabilized;

d) the balance is tared with the weighing device immersed in the liquid with the balance;

e) remove the counterweight and put the solid with its support on the weighing device;

f) record the value indicated on the scale to determine the apparent mass (M_L) made w hen the solid is immersed in liquid;

g) removing the solid and return the counterweight and record whether there were changes in the tares value.

2.4. Mathematical model for liquid density measurement

Once the volume standard is determined, an unknown liquid density "ρ_L"can be measured by using equation (4).

$$\rho_L = \frac{1}{V_r \cdot (1+\alpha \cdot (T_L - T_r))} \cdot \left(M_L \cdot \left(1 - \frac{\rho_a}{\rho_b}\right) - M_R \right) + \delta\rho_L + \delta\rho_S + \delta\rho_\sigma \qquad (4)$$

Where:

ρ_L is the liquid at temperature "T_L" [g cm^{-3}]

ρ_a is the air density during the balance calibration [g cm^{-3}]

ρ_b is the density of standard weighs those are used to calibrate the balance [g cm^{-3}]

$\delta\rho_L$ is the error due to the repeatability and reproducibility of liquid density measurement [g cm^{-3}]

$\delta\rho_S$ is the error due to volume difference between volume support and counterweight [g cm^{-3}]

$\delta\rho_\sigma$ is the error due to variation of liquid surface tension force on the wire [g cm^{-3}]

2.4. Procedures for liquid density measurement

The procedure is the same as for solid volume measurement. The difference is the kwon quantities values.

3. RESULTS

Table of solid pieces density and volume with their associated uncertainties.

Example of an uncertainty budget

Table 1. Volume measurement

VOLUME				
Samples	Measured value (cm^3)	Expanded uncertaint y (cm^3)	Relative exp. uncert.	Coverage factor k
1	17,19999	0,00079	0,0046%	2,010
2	15,50820	0,00092	0,0060%	2,047
3	15,47255	0,00094	0,0061%	2,057
4	4,28690	0,00060	0,014%	2,000
5	1,13074	0,00062	0,055%	2,000
6	0,89266	0,00061	0,068%	2,000
7	25,1575	0,0011	0,0040%	2,034

Table 2. Density measurement

DENSITY				
Samples	Measure d value (g/cm^3)	Expanded uncertainty (g/cm^3)	Relative exp. uncert.	Coverage factor k
1	7,97543	0,00037	0,0047%	2,010
2	7,96438	0,00047	0,0060%	2,047
3	7,96537	0,00049	0,0062%	2,057
4	7,9709	0,0012	0,015%	2,000
5	7,9729	0,0044	0,056%	2,000
6	7,9736	0,0054	0,068%	2,000
7	7,99753	0,00033	0,0040%	2,034

Observation: The coverage probability is approximately 95%.

The following tables 3 and 4show the error contribution due to the influence quantities for sample 1.

Table 3. – Volume uncertainty budget

Uncertainty budget of volume measurement			
Quantity	Expanded uncertainty	Coverage factor k	Relative uncertainty contribution $u_i(y)/y$
Mr	0.00044 g	2.000	23.2%
M_L	0.00035 g	2.000	18.4%
ρ_L	0.00001 g cm^{-3}	2.000	10.4%
T_L	0.021 ºC	1.732	3.9%
α	0.0000048 ºC^{-1}	1.732	6.9%
δV	0.00040 cm^3	2.3198	18.2%
δV_S	0.0000128 cm^3	1.732	0.8%
δV_σ	0.00030 cm^3	1.732	18.1%

Table 4. - Density uncertainty budget

Uncertainty budget of density measurement			
Quantity	Expanded uncertainty	Coverage factor k	Relative uncertainty contribution $u_i(y)/y$
Mr	0.0004 g	2.0000	6.6%
V_R	0.00080 cm^3	2.0104	93.4%

4. DISCUSSIONS

The samples 1 to 6 was from same type of stainless steel. The samples 2 and 3 was different parts of same bar, samples 5 and 6 was one piece that was cute in two. The samples 1 and 4 was from different bars.

The sample 7 was from other type of stainless steel.

All result of density measurement is coherent to the origin of samples.

The main contribution to the uncertainty for volume is the real mass, but it can be reduced if it is measured by a mass laboratory, where can have better condition, with a lower uncertainty:

The principal contribution to the uncertainty for density is the volume.

The contribution on volume relative uncertainty can be minimized using greater volume solid. It make possible get better result on density measurement.

5. CONCLUSION

A good result of density and volume measurement can be achieved using the pure water as reference material for density, .the model based on solid real mass and the simples hydrostatic weighing system.

The results was good enough to distinguish density of different type of stainless steel and variation between bars of same type.

The results shows that the hydrostatic system can provide good traceability for calibration of density meters, volume standard and flow meter.

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