INFLUENCE OF THE SAMPLING STRATEGY ON
FEED-FORWARD CONTROL

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Abstract: The Feed-Forward Control Model (FFCM) relies on statistical fundaments to determine the necessary adjustments that can help controlling a given process. These adjustments are meant to counteract the influence of different variation sources over the process. In order to succeed there are two important factors that have to be considered: the size of the Look-up Window (LuW) and the sampling strategy.

Key words: Dynamic Tolerances, Forward-Looking Control, Micro-assembly, Look-up Window.

1. INTRODUCTION

In the production environment, the principle of interchangeability has been considered for a long time as the cornerstone of the serial production. Under this principle, all parts have to be fabricated, up to certain point, identically. Therefore, great part of the effort is focused on meeting the specifications to make possible the assembly of new devices and the repair of the existing ones.

In the domain of micro-production, parts usually present complicated geometries and their fabrication requires sophisticated equipment and high-qualified labor. When a micro-device breaks down the only practical alternative to a costly and probably unpractical repair is to replace it by a new one. The elimination of the need for repairing microsystems offers a unique opportunity to explore innovative production methods for those processes whose final products are meant to be dischargeable.

2. TOLERANCE AND CAPABILITY INDEXES

Engineering tolerance can be seen as the range in which the values of a certain dimension are permitted to fluctuate. In the case of assemblies it is necessary to consider the tolerance of each part and the way in which they will be stacked.

Considering specific functional demands designers estimate the tolerances that the final assembled product needs to meet. In practice today often the assumption is made that all deviations might add in the worst possible way. Then the tolerances of each single part add up to give the tolerance of the assembly. If the tolerance of the assembly is given, this approach may lead to determine very tight tolerances for the single parts.

As an alternative, the statistical stacking or RSS (Root Sum Square) method it is assumed that most probably the deviations of the single parts cancel each other at least partially, because they are statistically independent. The relation of the tolerance of the assembly and the single part tolerances t_i is:

$$t_{stat} = \sqrt{\sum_{i=1}^{n} t_i^2}$$

(1)

Capability indexes are ratios used to compare the standard variation of a dimension of interest with the specification limits LSL (lower) and USL (upper) that define the tolerance band. The potential capability index C_p indicates the theoretical process capability of meeting the specification. If the mean μ of a given distribution is not on target, it is necessary to take the shift into account and then the actual capability index C_{pk} has to be used instead.

$$C_p = \frac{USL - LSL}{6\sigma}$$

(2)

$$C_{pk} = \min\left\{ \frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma} \right\}$$

(3)

High capability indexes will help ensure the viability of a process, technically and economically speaking. A process is commonly accepted as capable when its potential capability index (C_p) is higher then 1.33. A lower C_p would require the process to be reviewed and re-engineered. Nevertheless, in some cases the problem of a low C_p can be overcome by means of applying innovative techniques like the dynamic tolerances.

3. DYNAMIC TOLERANCES

The technique is based on the iterative application of the concepts behind the statistical stacking method to reduce the deviation resulting from combining processes that have a high variation.

Theoretically, the dynamic tolerance method relies on the statistical properties that govern the normal distributions

and the success of its application will depend on the type of variation of the distribution under study. The following sections will help understand this method by means of an example.

Let $L_{assy} \pm t_{assy}$ be the design specifications of an assembly made of two single parts: Component 1 and Component 2 whose nominal specifications are set to L_i and t_i.
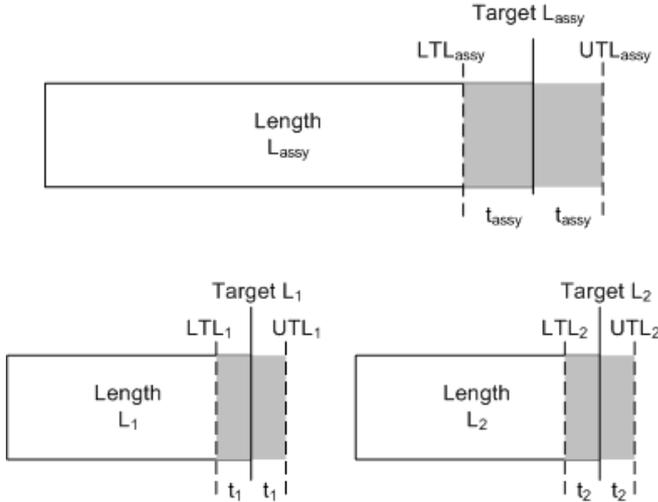

Fig. 1 Nominal Specifications

All the dimensions in the Fig.1 are related by the following equations:

$$L_{assy} = L_1 + L_2 \qquad (4)$$

$$t_{assy} = \sqrt{t_1^2 + t_2^2} \qquad (5)$$

The corresponding production processes of Component 1 and Component 2 are assumed to be characterized by two not correlated normal distributions. Namely $N_1(\mu_1,\sigma_1)$ and $N_2(\mu_2,\sigma_2)$.

The sum of two normally distributed variables will result in a new normally distributed variable that in this case will obey to the following equations:

$$\mu_{assy} = \mu_1 + \mu_2 \qquad (6)$$

$$\sigma_{assy} = \sqrt{\sigma_1^2 + \sigma_2^2} \qquad (7)$$

In any case the resulting distribution will have a higher standard deviation than those of the individual contributors as they are not correlated.

If the variation of a production process is conceived as the result of the action of different factors it is reasonable to think that it can be separated in at least to mayor components: a random component and a drift or long-term

effect. The following figure shows how this idea would look like in a sample of units of the Component 1
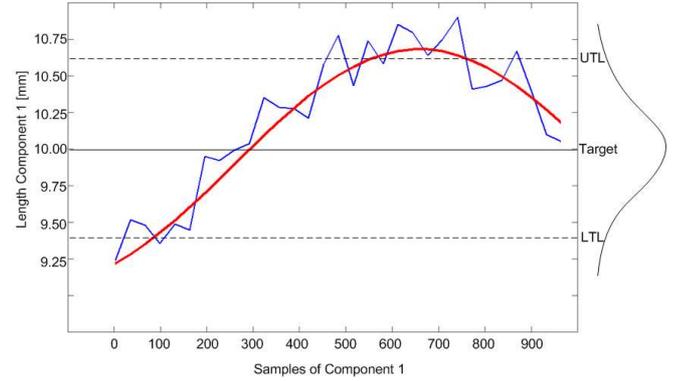

Fig. 2 Length Variation of Component 1

A process with a variation like this will have low capability index C_p and will certainly produce a great number of defective units as many of the samples lie out of the nominal tolerance limits. However, it is in this sort of situations where the dynamic tolerances approach could be a truly powerful tool.

The existence of the drift, or long-term variation pattern, gives the chance to analyze the sample not as whole but partially as to deal with its high variation. The following picture shows that if initially only a reduced sample is considered, the variation found in its vicinity would be lower than the one for the complete sample.
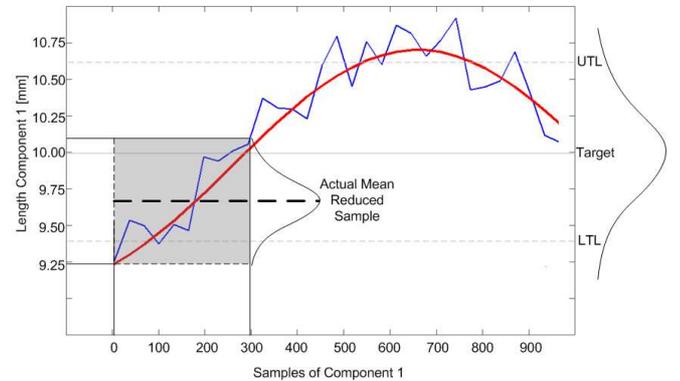

Fig. 3 Reduced Sample of Component 1

There are two immediate benefits of applying this technique. The first one is that the standard deviation of this reduced sample, $\sigma_{1,rs}$, is lower than the one of the whole sample σ_1. Thus, if it is combined with the distribution of the Component 2, the resulting distribution will also have a lower standard deviation.

$$\sigma_{1,rs} \leq \sigma_1 \qquad (8)$$

$$\sqrt{\sigma_{1,rs}^2 + \sigma_2^2} \leq \sqrt{\sigma_1^2 + \sigma_2^2} \qquad (9)$$

The second benefit is that it is possible to redefine completely the nominal specifications of the Component 2.

Given that the value of the actual mean of this reduced sample, $\mu_{1,rs}$, is known and presumable different from the original nominal specification L_1, the length L_2 of Component 2 can be adjusted as to match the desired specification L_{assy}.

$$L_{2,adj} = L_{assy} - \mu_{1,rs} \qquad (10)$$

Thereby, as the variation of the reduced sample of the Component 1 is lower that the variation for the complete sample, or equal in the worst case, then it is reasonable to think that the units within this reduce sample will hardly make use of the whole nominal tolerance defined for the Component 1 and that part of that tolerance band can be transferred to the specification of the Component 2.

From the Fig. 3 it can be seen that all the units of the reduced sample lie in a narrower tolerance band. In fact, if this reduced tolerance band is assumed to have a width equivalent to six times $\sigma_{1,rs}$ it would be enough to guaranty that 99.99% of the samples will lie within the band. Of course, the underlying assumption is that six times $\sigma_{1,rs}$ is still lower than two times t_1.

$$6\sigma_{1,rs} \leq 2t_1 \qquad (11)$$

$$LTL_{1,rs} = \mu_{1,rs} - 3\sigma_{1,rs} \qquad (12)$$

$$UTL_{1,rs} = \mu_{1,rs} + 3\sigma_{1,rs} \qquad (13)$$

$$t_{1,rs} = \frac{UTL_{1,rs} - LTL_{1,rs}}{2} = 3\sigma_{1,rs} \qquad (14)$$

In this way, the new nominal specification for the tolerance of the Component 2 can be calculated as follows:

$$t_{2,adj} = \sqrt{t_{assy}^2 - t_{1,rs}^2} \qquad (15)$$

$$t_2 \leq t_{2,adj} \qquad (16)$$

The direct consequence of having such extended tolerance, $t_{2,adj}$, is the increment of the capability of the process as it is shown in the equation 2. In other words, the number of units lying out of the extended tolerance band has to be lower and therefore the process will result in a smaller number of rejections or defective units. The Fig.3 presents graphically the concepts exposed above.
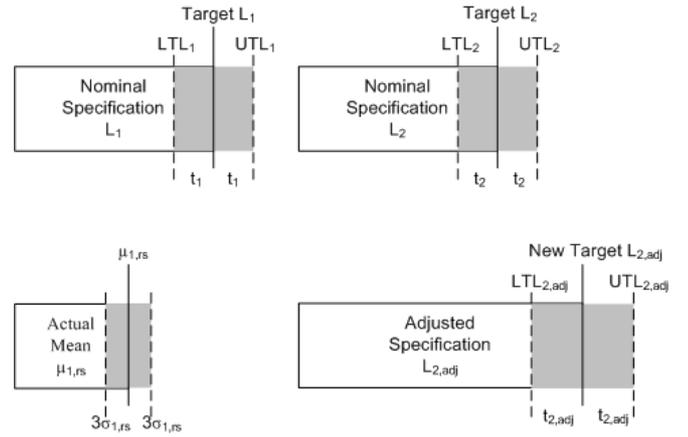

Fig. 3 Adjusted length for the Component 2

The same sort of analysis can be applied iteratively as many times as desired until the whole sample is covered. This is shown in the Fig.4.
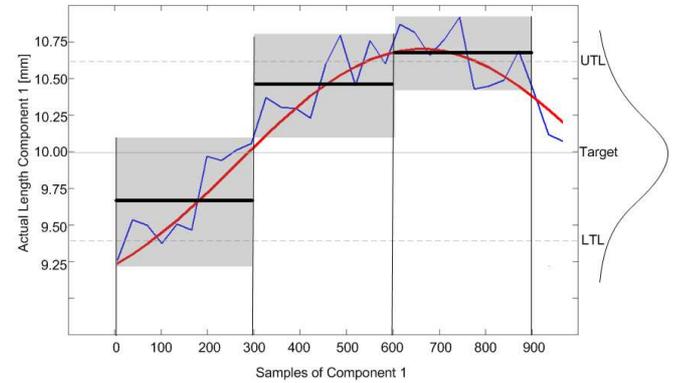

Fig. 4 Dynamic Tolerance Method

In the case of the distribution of the Component 1, neither the mean μ_1 nor the standard deviation σ_1 will suffer any change. Instead, the distribution of the Component 2 will present both a change in the mean μ_2 as a direct consequence of the iterative adjustments of its target and an increment in the standard deviation σ_2 due to the displacement of the adjusted tolerance band along with the moveable target. The values of the length in a common sample of the Component 2 are shown in the Fig. 5.
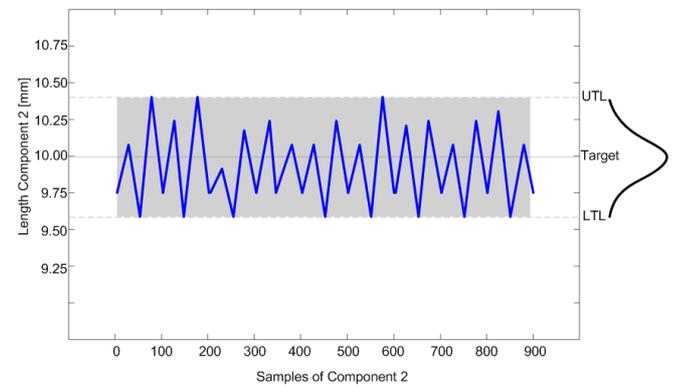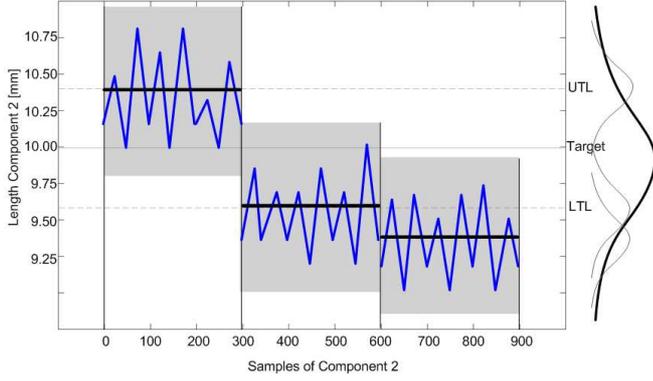

Fig. 5 Length Variation of Component 2

Fig. 6 Variation of Adjusted Specification of Component 2

As the target length of the Component 2 is modified dynamically, the resulting values will present a higher variation as it can be seen in the Fig. 6. The recursive application of the dynamic tolerance method over a sample, as shown in the Fig. 7, will lead to another important result.
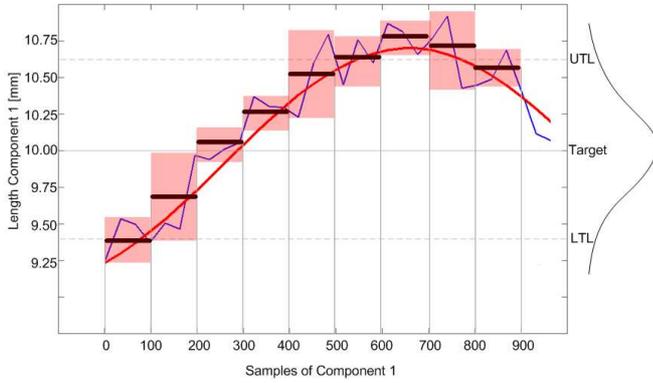
Fig. 7 Recursive Application of Dynamic Tolerances

Given that the target of the sample of Component 2 is adjusted after every iteration, its value will vary through out the production cycle depending on the actual mean of the reduce samples of Component 1. Therefore if both samples are analyzed not partially but completely, then it has to be accepted that their distributions are now correlated. Thus, the equations that relate these two distributions will look like slightly different.

$$\mu_{assy,adj} = \mu_1 + \mu_{2,adj} \tag{17}$$

$$\sigma_{assy,adj} = \sqrt{\sigma_1^2 + \sigma_{2,adj}^2 + 2\rho\sigma_1\sigma_{2,adj}} \tag{18}$$

In the equation 18, ρ represents the correlation coefficient between the distributions. In this case, it is not hard to see that they are inversely correlated as the adjustments in the target of the Component 2 are meant to compensate the deficit or excess detected in the actual mean of the reduced samples of the Component 1. Therefore the result of combining these two distributions will reflect the iterative compensation effect of the dynamic specifications that it expected to produce a reduced standard deviation.

$$\sigma_{assy,adj} \leq \sigma_{assy} \tag{19}$$

The consequences of applying the dynamic tolerance method are not minor. Firstly, it could help to improve the capability of a process without re-engineering it. And secondly, it would make possible to reduce the number of rejections thanks to the extended tolerance bands because there would not be any need for rejecting a unit that could actually lie out of the designed nominal tolerance band.

Undoubtedly the most remarkable result of dynamic tolerances is the flexibility offered by the adjusted specification and the extended tolerances. In this example, the Component 2 will end up with a higher capability index granted by the extended tolerance zone. However, these advantages do not come along for free. In a production environment, this method would require a significant increment in the number of the units to measure and in the adjustments to be made. To overcome this obstacle, the innovative approach of feed-forward control will be presented in the next sections.

4. FEED-FORWARD CONTROL MODEL (FFCM)

There are some particular characteristics that let FFCM apart from the traditional control approaches. Perhaps the most important and necessary of them is the identification and definition of two sub-systems within the system under control. The sub-systems must be defined in such a way that the system can be split into two parts as to make possible the introduction of an additional measurement between them to retrieve data about the actual values of the system variables.
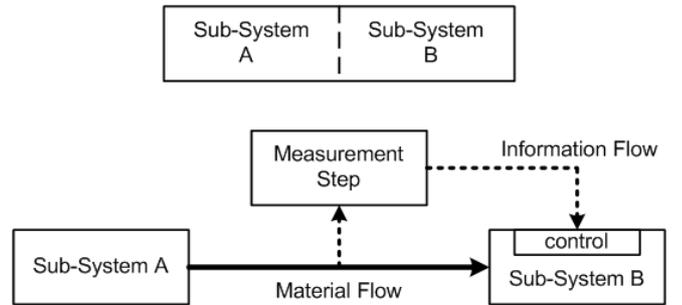
Fig. 8 Sub-systems A, B and the additional measurement step

Being able to retrieve the actual values of the output variables of the feeding sub-system A is an indispensable condition due to the fact it is the only way to apply corrective countermeasures during the inner work of the controlled sub-system B to ensure that the final output is as close as possible to the desired one. The measurement does not have to cover 100% of the output of process A but it can be reduced to samples, similar to a classical SPC.

Whereas in the closed-loop controller model the system is considered like a black-box whose inputs can be adjusted as to diminish the difference between the output and the reference; in the FFCM the black-box is replaced by two sub-systems whose output and input are measured and adjusted respectively.

FFCM aims to reduce the variation of a process and to produce a better output by applying correcting countermeasures from the inside of the system. This implies that some of the system's internal parameters will vary in time in order to generate the combined effect that will accommodate the output in such a way that its difference with respect to the reference is minimized.

5. SIMULATION

5.1 Initial Assumptions

In the following section we present the results obtained after the simulation of a micro-assembly fabrication as to show all concepts described so far. The assembly will consist of three components, namely 1, 2 and 3, whose specifications are identical and which are produced by Process 1, 2 and 3 respectively. For simplicity, all the considerations related to the uncertainty of the measured values resulting from the simulation will be left apart.

Before proceeding, it is necessary to state several assumptions that will be taken as valid:
· **Normality**. All of the components' samples obey to normal distributions that are characterized by a certain mean and a standard deviation.
· **No correlation**. Process 1, 2 and 3 belong to different production lines. Thus, there is not reason to consider any correlation among them.
· **Process variation**. Similar to SPC we assume the statistical variations of the processes to be separable into a short term noise (e.g. caused by mechanical vibration) that cannot be controlled at all and a variation on la longer time scale (e.g. caused by temperature variation) that are potentially controllable.

5.2 Design Specifications

For the purpose of this exercise, a lot of one thousand micro-assemblies with the following specifications will be simulated. The length is the dimension of interest.

Once the specification of the assembly has been decided, it is possible to proceed with the design of the dimensional specifications of the corresponding components.

Table 1 Assembly and components specifications

	Length [mm]	Tolerance [mm]
Assembly	30.00	1.00
Component 1	10.00	0.58
Component 2	10.00	0.58
Component 3	10.00	0.58

Whereas the length of each component can be estimated with relatively little effort, the corresponding value for their tolerances is not really obvious and demands the application of the equation 1 to be understood.

5.3 Process Characterization

Perhaps the most important of the initial assumptions is that one related to the normality of the probability distributions characterizing the production processes of Process 1, 2 and 3. The following table summarized these characteristics and the corresponding indexes C_p. Obviously in this simulation all these processes are not capable.

Table 2 Process characterization and capability index

	Mean [mm]	St. Dev. [mm]	C_p
Process 1	9.75	0.15	1.29
Process 2	9.85	0.20	0.97
Process 3	9.95	0.15	1.29

6. ASSEMBLING COMPONENTS

As it was established in the initial assumptions, Components 1, 2 and 3 are fabricated in different production lines and consequently their samples can be considered as not correlated random variables.

The following table presents the mean and the standard deviation resulting from combining the probability distributions of Components 1, 2 and 3. The read of these results is rather simple, a lot of thousand assemblies is expected to be characterized by N(29.55 , 0.30).

Table 3 Resulting probability distribution

	Mean [mm]	St. Dev. [mm]	Cp
Assembly	29.55	0.30	1.13

In this case the resulting distribution will always have a higher standard deviation which means that the complete process or system will have always a joint variation that is higher than those ones of the individual contributors.

The sub-assembly of Components 1 and 2 will be taken as the feeding sub-system A, while the controlled sub-system B is given by the Process 3, manufacturing Component 3. Thus, the challenge is finding a proper and feasible way to adjust the input to the sub-system B with the help of the data retrieved from the sub-system A. In this case the parameters to adjust are nothing else but the specifications of Component 3.

7. LOOK-UP WINDOW

The look-up window is meant to define the number of units taken from the output of the sub-system A that will be considered at once to estimate the adjustment needed for the input to the sub-system B. Not all of these units are expected to be measured.

For the simplicity of this exercise and to have always a minimum of ten adjustments, in this case the look-up window will be defined to be equal to a tenth of the production lot. As the lot size was arbitrarily defined to be one thousand, the look-up window will be set to be one hundred.

7.1 Look-up Window Sample

A crucial question is to determine how many measurements are necessary to obtain a representative value of the LuW's content. The selection of the units to measure will be random.

With the help of a customized algorithm to reduce noise and to interpolate points, the complete content of the look-up window will be reconstructed. The algorithm was developed in such a way that all the historical data corresponding to the previous LuWs is considered to reconstruct the content of the current one as to help detecting eventual long time variation in the complete lot. Thus, the specifications for the Component 3 will be adjusted according to the actual representative value of the last LuW,

8. SAMPLE SIZE

Despite of the complexity of defining an adequate sampling strategy the underlying idea is rather simple, selecting some samples from a lot as to get a representative description. Intuitively, selecting and measuring as many samples as possible could be the easiest way to achieve an accurate representation of the lot. However, it would not be cost efficient and perhaps not even practicable. Then, the challenge strives in determining the appropriate number of samples that gives a value that represents satisfactorily the interval covered by the LuW.

In the Table 4 the results obtained after simulating the application of FFCM to control a micro-assembly process are given. The assembly length specification is 30.00 ± 1.00 [mm]. The values of the mean and standard deviation obtained before applying FFCM are $\mu=29.55$ and $\sigma=0.29$ respectively. Whereas the size of the LuW was set to one hundred, the number of samples per LuW was set to different values as to see its influence in the output of the process.

Table 4. Comparison among different sample settings

$Sample_{size}$	μ_{cntrl}	σ_{cntrl}	$\mu_{shift\ reduction}$	$\sigma_{improvement}$
5	29.9732	0.2703	6.34 %	6.79 %
10	29.9525	0.2594	11.80 %	10.55 %
15	29.9512	0.2565	12.16 %	11.55 %
20	29.9506	0.2525	12.33 %	12.93 %
25	29.9493	0.2545	12.70 %	12.25 %
30	29.9486	0.2559	12.89 %	11.76 %
40	29.9530	0.2561	11.65 %	11.70 %
50	29.9569	0.2560	10.59 %	11.74 %
75	29.9497	0.2523	12.60 %	13.01 %

As the number of measured samples increases from five to twenty a reduction of both the mean shift and the standard deviation can be achieved. However after that, despite of the significant increment in the number of measured samples there is not a definitive improvement of the output.

9. COMPARISON OF HISTOGRAMS

To realize in a better way the impact of applying FFCM it is useful to compare both the original and the adjusted histogram. In the following figure the curve on the right represents the distribution after applying FFCM with a LuW size of one hundred and sample size of ten units per LuW.
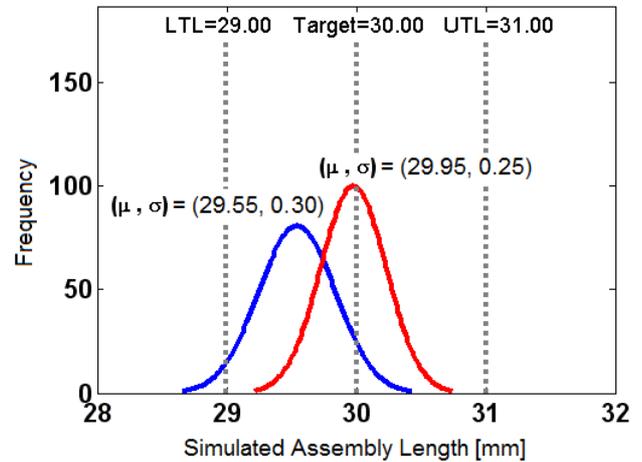
Fig. 9 Assembly – Histogram – Original (left) and controlled (right)

10. CONCLUSION

In fact, the number of measurements has an impact in the result of the control. However, measuring more units per LuW will not lead to a better control. In this simulation, measuring only 10% percent of the units was enough to produce a significant improvement in the output.

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