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A Four Terminal-Pair Coaxial Impedance Bridge Constructed at Inmetro

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Abstract: This paper describes the final construction stage and preliminary measurements results of a low-frequency four terminal-pair coaxial impedance bridge, part of the traceability chain of the capacitance unit to the quantum Hall effect, constructed at Inmetro. The coaxial bridge is a complex system, with several technical advantages, as high-stability and very low-uncertainty.

Key words: Coaxial current bridges, electrical metrology, impedance measurements.

1. INTRODUCTION

There are two main motivations for the construction of a four-terminal coaxial bridge at Inmetro. The first one is the implementation of the traceability chain of the capacitance unit to the quantum Hall effect [1]-[2]. The Inmetro quantum Hall effect (QHE), acquired recently, is already in operation. Preliminary measurements are being made in order to reduce random effects and to estimate its uncertainties [3].

The realization of the capacitance unit (farad) is related to the QHE by a calculable coaxial resistor, responsible for the DC/AC relation, and three low-uncertainty coaxial bridges: the two-terminal bridge, the four-terminal bridge, and quadrature bridge.

The two-terminal coaxial bridge [4]-[7] is already in operation at Inmetro, applied to the comparison of high-values impedances, with relative uncertainty of 2,0 x 10^{-8}, considering 1:1 comparisons and 1,0 x 10^{-7} considering 1:10 comparisons. The quadrature bridge, applied to the comparison of standard capacitors with AC resistors, is in the final stage of construction and should be in operation in 2012. We are constructing a coaxial calculable resistor of 1 kΩ, which will allow us to establish the frequency dependency of our AC standard resistors.

The second motivation for the construction of a four-terminal coaxial impedance bridge is to guarantee the traceability to the calibration of AC resistance and high-value capacitance standards. In the last few years, Inmetro has been receiving several requests to calibrate AC standard resistors and shunts, from laboratories, industries, and universities. The construction of the four-terminal bridge will allow our laboratory to assure the traceability for measurements of low-value impedances, in frequencies between 400 Hz and 3 kHz.

In order to construct a four terminal-pair coaxial bridge with high-stability and low-uncertainty, our laboratory established a partnership with others NMIs, as the *Laboratoire National of Metrologie et Essais* (LNE). Our bridge is based on the LNE design [8], with modifications at the injection system. We also considered the BIPM four-terminal bridge design [9].

In the following section we present a description of the four-terminal coaxial bridge. In the third section we describe the coaxial bridge main transformers and in the forth section we present preliminary measurement results and uncertainty estimations.

2. FOUR TERMINAL-PAIR COAXIAL BRIDGE

In order to be part of the traceability chain of the capacitance unit to QHE it is necessary to built a bridge with uncertainty of a few parts in 10^{-8}. To archive the desired low-uncertainty we decided to construct a coaxial four terminal-pair bridge [10]-[12], with the technical support of Dr. Alexandre Bounouh, from LNE-FRANCE and the financial support of Inmetro and Brazilian Government - FINEP. This bridge can be applied both for calibration purposes and to implement the capacitance traceability chain.

The coaxial bridge is a complex system, but it has several technical advantages, as high-stability, very low-uncertainty, and isolation from external noise sources. In the next subsections we detail the coaxial bridge layout, main and auxiliary balances, and its injection.

2.1. Main Balance

A schematic circuit of the coaxial four-terminal bridge is shown in Figure 1. The main balance of the coaxial bridge uses a two-stage inductive voltage divider (IVD) to provide the voltage ratio between the two impedances under comparison. We will present a description of the IVD in section 3.

We decided to use a passive injection for practical reasons, since we had technical difficulties to construct the active injection electronic circuits. We also had previous experience with the use of a passive injection circuit at Inmetro's two terminal-pair coaxial bridge [4]-[7], with good calibration results. We will describe the passive injection circuit and its equations in subsection 2.3. The output signal of the main balance is detected with a low-noise pre-amplifier and then it is applied to a DSP lock-in amplifier that operates as a null detector.

2.2. Auxiliary Balances

Besides the main balance, the four-terminal coaxial bridge has three auxiliary balances, [13]. The compensation

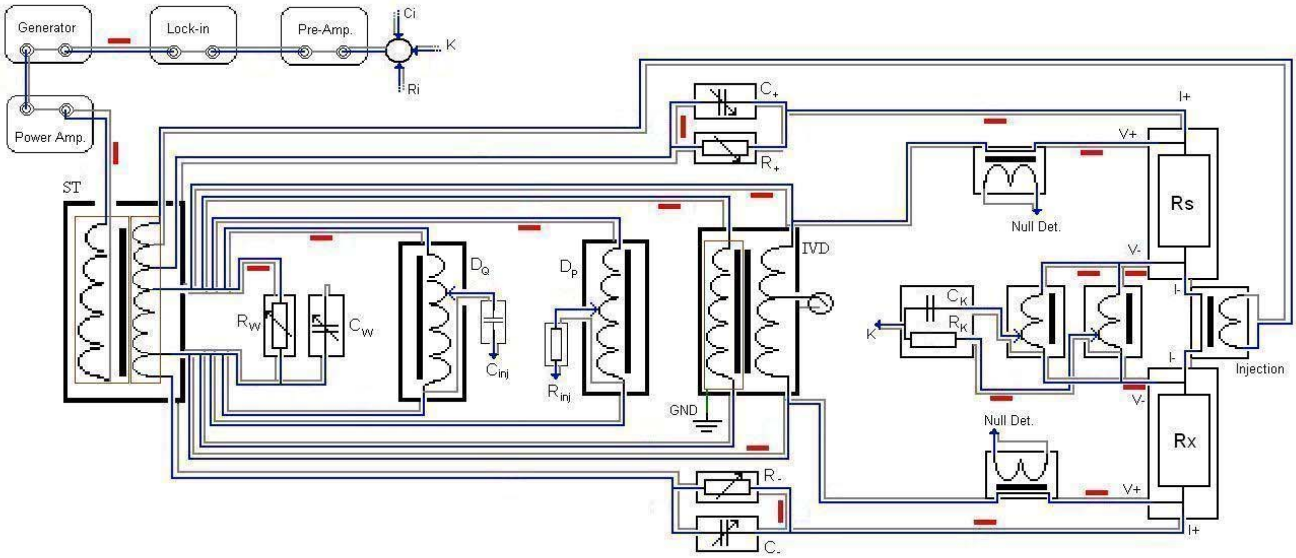

Fig. 1 Four-pair Terminal Coaxial Bridge

balance guarantees that there is no current in the high-voltage terminals of the impedances under comparison, compensating for cables impedances. These impedances are significant in low-resistance and high-capacitance standards comparisons. There are two separate arms for the compensation balance, at R_S and R_X high-voltage terminals; each arm has a decade resistor (R_+), a decade capacitor (C_+), and a 1:100 transformer.

The Kelvin balance allows the four terminal-pair conditions to be met at the low-voltage terminals of R_S and R_X. This balance also has two standard seven-decade inductive dividers associated with high-stability fixed resistor (R_K) and capacitor (C_K), as shown in Figure 1. The third auxiliary balance, Wagner, is necessary to assure that the IVD "central" tap is actually at zero potential, with no parasitic current flow. The Wagner balance is archived by a decade resistor (R_W) and a decade capacitor (C_W).

2.3. Passive Injection

The IVD supplies both standards impedances, R_S and R_X, with voltages of similar magnitude and a phase shift of π. To compensate for deviations from the nominal values of R_S and R_X, we inject variables currents that alter the IVD ratio, allowing for a zero voltage at the null detector [1]-[2], [13].

The in-phase current is injected by means of the injection resistor R_{inj} and the seven-decade inductive divider D_P, which provides an adjustable voltage. The quadrature current is injected by means of the injection capacitor C_{inj} and the inductive divider D_Q.

The main balance is archived when the voltage at the null detector is approximately zero. When comparing similar nominal value impedances, i.e., a 1:1 ratio comparison, the in-phase balance equation can be approximated by

$$\frac{1}{R_x} = \frac{1}{R_s} + \frac{(\alpha_1 - \alpha_2)}{R_{inj}} + \frac{(\beta_1 - \beta_2)}{R_{Cinj}} \qquad (1)$$

where the factors α and β correspond the settings of the inductive dividers D_P and D_Q, respectively. α_1 and β_1 are measured with R_S and R_X positioned as shown in Figure 1, and α_2 and β_2 are measured with R_S and R_X interchanged, thus far eliminating the IVD 1:1 ratio error. R_{inj} is injection resistor and R_{Cinj} is the parasite resistance of C_{inj}.

The four terminal-pair coaxial bridge can be very easily adapted to compare different nominal value impedances, with a simple adjustment of the IVD cables position. Considering a 1:10 ratio comparison, the in-phase balance equation can be approximated by

$$\frac{1}{R_x} \approx \frac{1}{1-D}\left(\frac{D}{R_s} - \frac{\alpha}{R_{inj}} - \frac{\beta}{R_{Cinj}}\right) \qquad (2)$$

where the factors α and β are measured with R_S and R_X interchanged. R_{inj} is injection resistor, R_{Cinj} is the parasite resistance of C_{inj}, $D = 1/11+k1$, and $k1$ is the IVD in-phase ratio error.

For a 1:10 ratio comparison the IVD supplies the standards impedances, R_S and R_X, with different voltages, where the voltage in R_X is 10 times the voltage in R_S. Since it is not possible to interchange R_S and R_X in 1:10 ratio comparison, it is necessary to obtain the IVD ratio error, as described in section 3.

The passive injection circuit and its equations, described above, refer to the comparison between two standard AC resistors. But the passive injection can be adapted to allow the comparison of two standard capacitors.

3. INDUCTIVE VOLTAGE DIVIDER AUTO-TRANSFORMER

An essential part of four terminal-pair coaxial is a two-stage inductive voltage divider auto-transformer - IVD. Due to the construction method employed here, the IVD allows the comparison of standard impedances with different nominal values. This type of auto-transformer is very stable and presents a highly accurate ratio, for both 1:1 and 1:10

comparisons. Our IVD was constructed partially based on the LNE design [8]. Besides the IVD auto-transformer, the four terminal-pair coaxial bridge has four additional transformers, the source transformer (ST), two detection transformers, and an injection transformer.

3.1. IVD Calibration

It is necessary to know the gain relations of the IVD when calibrating impedances with different nominal values. Due the construction method employed with the IVD, its gain can be estimated, considering only the number of turns, with an uncertainty of parts in 10^{-6}. Since this value is still too high for our applications, it was necessary to calibrate the IVD ratio errors.

Initially we calibrated the IVD in-phase ratio error by comparison with standard capacitors [2]. This calibration was performed using two known silica standard capacitors of 10 pF and 100 pF, and the two terminal-pair coaxial bridge [5], [6]. Both standard capacitors where calibrated at BIPM through QHE. Although this method presents a higher uncertainty, it is a simple and fast method that was already available to us.

We calibrated the IVD in-phase ratio error $k1 = 2{,}0 \times 10^{-8}$ at 1 kHz, with uncertainty $u = 4{,}0 \times 10^{-8}$. The value of $k1$ can be considered very small, showing the effectiveness of the IVD design. Due to the calibration method limitations it was not possible to calculate the IVD quadrature ratio error $k2$. Considering the construction methods employed here, we can assume that $k2$ is very small, [2].

4. PRELIMINARY MEASUREMENTS

In order to verify the four-pair of terminal bridge design and to confirm its efficiency we made several preliminary measurements. We compared two standard AC resistors with nominal value of 1 kΩ, and also a 1 kΩ resistor with a 10 kΩ resistor, both at a frequency of 1 kHz.

The 1 kΩ standard resistor, here referred as R_S, was calibrated both for its DC value and to its frequency dependence. The latter calibration was realized at the LNE low-frequency laboratory. The second 1 kΩ resistor, here referred as R_{X1}, was calibrated only for its DC value at Inmetro DC resistance laboratory. The 10 kΩ standard resistor, here referred as R_{X2}, was also calibrated both for its DC value and to its frequency dependence.

Initially we made measurements with the bridge in the configuration 1:1, as shown in Figure 1, in order to compare R_S and R_{X1}. After a series of five measurements, we were able to measure the impedance of R_{X1} at 1 kHz, with a relative standard deviation of 0.01 μΩ/Ω. We consider this result adequate to our purposes of obtaining an uncertainty of a few parts in 10^{-8}.

We also made preliminary measurements with the bridge in the configuration 1:10, in order to compare R_S and R_{X2}. After a series of five measurements, we measured the impedance of R_{X2}, also at 1 kHz, with a relative standard deviation of approximately 0.1 μΩ/Ω. We consider this an acceptable result; although we expect to improve the measurement repeatability with a few minor additional adjusts at the coaxial bridge 1:10 configuration. The following subsections describe the preliminary uncertainty calculations of the coaxial bridge for 1:1 and 1:10 configurations.

4.1. Uncertainty Evaluation for 1:1 Configuration

We first evaluated the uncertainty associated with the measurement result for the 1:1 ratio comparison between similar standard resistors. Table 1 shows the contributions to the uncertainty budget for the 1:1 calibration of a standard AC resistor of 1 kΩ (R_{X1}) from a similar resistor (R_S) traceable to BIPM and to LNE. The main contributions at the uncertainty budget are due to the uncertainty reported by the R_S calibration certificate (DC value and frequency dependence) and to measurement (α) repeatability.

TABLE I
UNCERTAINTY BUDGET (1 kΩ – 1 kHz)

Quantity	**Uncertainty (kΩ)**	**Type**
R_S	5.3×10^{-7}	B
α	5.2×10^{-9}	A
β	4.1×10^{-10}	A
R_{inj}	6.7×10^{-11}	B
R_{Cinj}	5.9×10^{-10}	B
u_{RX1}	**5.3×10^{-7}**	**Comb.**

We can observe in Table 1 that the main issue with the combined R_{X1} uncertainty is the R_S calibration uncertainty. The uncertainty u_{RS} = 0.5 μΩ/Ω is due to the fact that Inmetro QHE is not yet in operation. However, in the next few months we will be able to calibrate R_S directly from Inmetro QHE, which will allow us to get the current uncertainty reduced tenfold.

If we consider only the uncertainty contribution due to the coaxial bridge, the most significant uncertainty factor is due to α (in-phase injection) repeatability. Other contributions to the uncertainty budget are due to the injection passive components (R_{inj} and R_{Cinj}) and quadrature injection repeatability (β). The total contribution to uncertainty budget, if we do not consider R_S calibration uncertainty, is less than one part in 10^{-8}, which will enable the four-terminal pair coaxial bridge to be part of the traceability chain of the capacitance unit to QHE.

Although at the traceability chain of the capacitance unit to QHE the coaxial bridge will mostly be operated at the 1:10 configuration, it's also very important to operate the bridge at the 1:1 configuration, in order to obtain the standard resistances frequency dependency. We are now constructing at Inmetro a coaxial calculable resistor of 1 kΩ, which will allow us to establish the frequency dependency.

4.2. Uncertainty Evaluation for 1:10 Configuration

We have made preliminary measurements comparing the 1 kΩ standard resistor R_S with the 10 kΩ standard resistor R_{X2} at 1 kHz. Table 2 shows the contributions to the uncertainty budget for this calibration, with the bridge at 1:10 configuration.

We can also observe in Table 2 that the main issue with the combined R_{X2} uncertainty is the R_S calibration uncertainty, of u_{RS} = 0.5 μΩ/Ω.

TABLE 2
UNCERTAINTY BUDGET (10 kΩ – 1 kHz)

Quantity	Uncertainty (kΩ)	Type
R_S	5.3×10^{-6}	B
α	6.6×10^{-7}	A
β	1.4×10^{-9}	A
R_{inj}	2.1×10^{-9}	B
R_{Cinj}	1.7×10^{-8}	B
k1	2.1×10^{-7}	B
u_{RX1}	**5.3×10^{-7}**	**Comb.**

If we consider only the uncertainty contribution due to the coaxial bridge, the most significant uncertainty factors are due to α (in-phase injection) repeatability and IVD in-phase ratio error *k1*. The total contribution to uncertainty budget, if we do not consider R_S calibration uncertainty, is of 0.07 μΩ/Ω, which is close to our objective.

5. CONCLUSION

The Capacitance and Inductance Laboratory at Inmetro developed a four terminal-pair coaxial bridge, part of traceability chain of the capacitance unit to QHE. Preliminary results shows that the coaxial bridge is capable to operate with an uncertainty of parts in 10^{-8}, which confirms the coaxial bridge as the low-uncertainty system desired as part of a traceability chain.

Future developments, as the construction of the calibration transformer and the coaxial calculable resistor, will allow the bridge to operate at its full potential, as part of the system that will reproduce the capacitance unit at Inmetro.

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