



REALIZATION OF THE INDUCTANCE STANDARD AT CEM

*Y. A. Sanmamed*¹, *A. Sánchez*², *K. Schweiger*³, *J. Díaz de Aguilar*⁴, *R. Caballero*⁵, *M. Neira*⁶

¹ CEM (Centro Español de Metrología), Tres Cantos, Spain, yalvarezs@cem.mityc.es

² CEM (Centro Español de Metrología), Tres Cantos, Spain, asanchez_gar@yahoo.com

³ CEM (Centro Español de Metrología), Tres Cantos, Spain, kschweiger@cem.mityc.es

⁴ CEM (Centro Español de Metrología), Tres Cantos, Spain, jdiaz@cem.mityc.es

⁵ CEM (Centro Español de Metrología), Tres Cantos, Spain, rcaballero@cem.mityc.es

⁶ CEM (Centro Español de Metrología), Tres Cantos, Spain, mneirar@cem.mityc.es

Abstract: Inductance has been realized at Spanish Centre of Metrology (CEM) by means of a modified coaxial Maxwell-Wien bridge. This method ensures inductance traceability from the SI unit of capacitance and establishes the value of the CEM's set of 10 mH standard inductors at the frequency of 1 kHz.

Key words: Inductance, capacitance, resistance, impedance bridge.

1. INTRODUCTION

The henry is a SI derived unit for inductance. Until the development of the calculable capacitor [1], the henry was realized by an inductance standard geometrically calculable [2]. From the henry were derived the farad and the ohm by means of ac bridges [3, 4]. Nowadays, the inductance is linked to the capacitance using appropriate ac bridges. The most employed AC bridges are the resonance bridge [5, 6] and the Maxwell-Wien bridge [7, 8, 9].

In the resonance method inductance is traceable from the units of frequency, capacitance and resistance. This kind of bridge is usually employed in order to compare two impedances of the same type. Instead of two like impedances, a series or parallel resonance circuit containing an inductance could be connected in parallel with a variable capacitor and these two elements are balanced against a resistance standard. The resonance frequency is reached by tuning the variable capacitor, and then the bridge is balanced using the resistors.

According to the Maxwell-Wien bridge, the unit of inductance is derived from ac resistance and capacitance. Such bridge is a type of Wheatstone bridge used for measuring an unknown inductance in terms of calibrated resistance and capacitance. The bridge balancing consists in an appropriate selection of the bridge values in order to reduce the differential voltage to zero. Maxwell-Wien bridge is not frequency dependent.

Another method to measure inductance is based on the three-voltage method [10], which permits the comparison of unlike impedances. The principle of the method is based on the relation of impedances modules and phases. Its accuracy depends mainly of modern ac voltmeters. The main elements of the circuit are the standard impedance and the impedance to be calibrated connected in series. The voltage

dropped in each impedance and the total voltage are measured with a high accuracy ac voltmeter. The unknown impedance is obtained as a function of standard impedance and the three measured voltages. By this method, inductance is obtained by means of ac resistance as standard impedance.

More recently, bridges based on automatic sampling systems have been developed [11]. They consist on a comparison between an inductance and a reference resistor by means of two analog-to-digital converters. Such converters allow the determination of the current through the two standards. When the balance is achieved, the impedance ratio is given by the ratio of the measured voltage.

In order to obtain the best accuracy and based on our acquired experience in the development of coaxial quadrature and voltage ratio bridges [12], the Maxwell-Wien bridge was decided to be the most suitable method for CEM.

The main advantage of the ac bridges (i.e., resonance and Maxwell-Wien) is their low uncertainty. Besides, the Maxwell-Wien bridge is independent of frequency, which means that the bridge could be operated in a large frequency range. On the other hand, digitally assisted bridges do not provide as high accuracy measurements as the above mentioned ac bridges. In order to obtain small measurement errors, especially at high frequencies, amplifiers with very good dynamic properties should be employed. However, such bridges are automated and less time is spent in calibration.

This paper describes the basis procedure and the experimental set-up employed at CEM for the establishment of the unit of inductance based on the Maxwell-Wien bridge.

The final aim of this work is the implementation of the National Inductance Standard at CEM [13].

2. TRACEABILITY

The CEM traceability chain for inductance realization is based on Quantum Hall Effect [14] and a Josephson potentiometer to obtain the dc resistance. In order to link the dc resistance to ac resistance a calculable resistor with a determined ac-dc difference is employed. The ac resistance

is the starting point of the capacitance realization. That is obtained by means of four terminal-pair ac coaxial bridges (to compare like impedance standards) and a quadrature bridge to compare capacitors with resistances). As long as this system is not validated, traceability of the farad at CEM is assured through a transfer standard calibrated at BIPM. Thus, the realization of the henry is achieved from the capacitance and the ac resistance. Figure 1 shows schematically the inductance traceability chain.

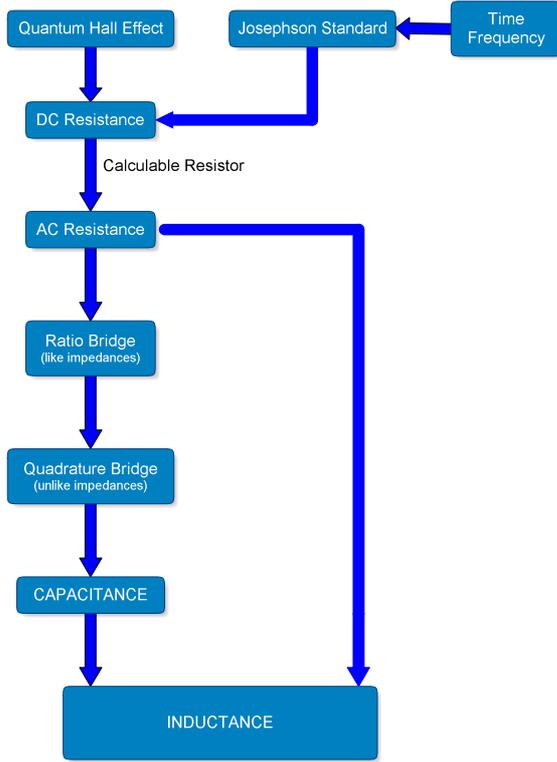


Fig. 1. Traceability chain for the inductance realization at CEM.

2. MAXWELL-WIEN BRIDGE AT CEM

The Maxwell-Wien bridge, was originally developed by J.C. Maxwell for ballistic purposes and adapted by M. Wien for ac measurements. In this case, it is employed in order to determine the inductance value in terms of a well-known capacitance and two resistances.

In figure 2 the Maxwell-Wien bridge implemented at CEM is shown. For simplicity, the outer conductor of the coaxial bridge is not represented. The main elements of the bridge are the inductance to be calibrated, represented by the series equivalent inductance and resistance of the standard (L_x and R_x , respectively), a 10 nF standard capacitor C_s , a precise variable air capacitor C_v , two fixed resistors R_1 and R_2 and one variable resistor R_v .

The inductance is a General Radio GR-1482, it mainly consists on a uniformly wound toroid on a ceramic core. It has a negligible external magnetic field and hence essentially no pickup from external fields.

The standard capacitor C_s selected is a GR-1409-L highly stable, hermetically sealed with a ceramic dielectric and a fixed nominal value of 10 nF.

The fixed resistors R_1 and R_2 are 1 k Ω hermetically sealed resistances obtained from Vishay. They were chosen for their good stability, small temperature coefficient ($0.6 \cdot 10^{-6}/^\circ\text{C}$), non inductivity and low ac-dc difference.

Finally, R_v is a combination of two 50 k Ω Vishay resistors and three multiturn potentiometers, used to make fine adjustment between 100 k Ω and 120 k Ω . Furthermore, two switches in the box containing R_v allow the connection of two capacitors. This T-circuit generates a negative capacitance to balance the bridge if it is necessary.

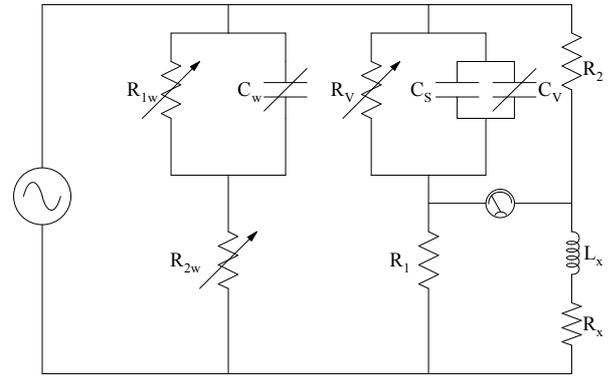


Fig. 2. Circuit of the Maxwell-Wien bridge with the Wagner arm.

To enable the connection of all the bridge components a metallic shielded box was designed and constructed, minimizing the residuals parameters. All connections in the box are made by means of British Post Office (BPO) connectors. Resistors R_1 and R_2 are screened and placed in separate compartments of the box.

A general view of the Maxwell-Wien bridge at CEM is shown in figure 3.

When the Maxwell-Wien bridge is balanced the inductance standard and its internal resistance are calculated according to equations (1) and (2).

$$L_x = CR_1R_2 \quad (1)$$

$$R_x = \frac{R_v R_1}{R_2} \quad (2)$$

The main bridge balance is achieved with components R_v and C_v . In order to avoid errors in the circuit resulting from leakage impedances an auxiliary Wagner arm is employed. The components used for the Wagner branch are two variable resistors (R_{1w} and R_{2w}) and a variable capacitor (C_{1w}). Thus, the bridge is balanced simultaneously by means of the main and the Wagner balances.



Fig. 3. General view of the Maxwell-Wien bridge.

To eliminate stray capacitance between bridge nodal points and ground and the stray inductance on the inductive coil a zero-substitution method [15] is used. Inductor L_x is replaced by a small self-made inductor, L_{x0} , which has an adjustable resistor R_{x0} . This resistor can be fixed at the same nominal resistance as the intrinsic resistance of the unknown standard inductor L_x . The capacitor C_s is removed from the bridge for the auxiliary measurement.

The unknown inductance L_x can be obtained from (3):

$$L_x = R_1 R_2 (C - C') + L_{x0} \quad (3)$$

where C is the capacitance in the bridge when the unknown inductor L_x is connected, C' is the capacitance resulting when the small inductor L_{x0} is measured instead of L_x and L_{x0} is the value of the small inductor.

The inductance scale is maintained at the rated value 10 mH, realized by a set of standard inductors, which are uniformly wound toroids on a ceramic core, having a stability better than $100 \cdot 10^{-6}$ per year and low temperature coefficient. In order to reduce the uncertainty due to inductor temperature's coefficient they are maintained in an air thermostat.

3. UNCERTAINTY BUDGET

Evaluation of measurement uncertainties has been carried out according to GUM [16] and the uncertainty budget for the realization of the 10 mH inductance standard at 1 kHz is shown in table 1. Using the law of propagation of uncertainty, the relative expanded uncertainty of the 10 mH inductance has been estimated at $30 \mu\text{H}/\text{H}$ for $k=2$.

Commercial inductance standards have a large temperature coefficient. In order to reduce the uncertainty due to temperature coefficient, they should be thermally stabilized or their internal temperature accurately measured. For the internal temperature determination the dc resistance of the inductance is used as a thermistor. In order to apply the temperature corrections, the relation of the temperature, resistance and inductance variations are determined.

The inductors are immersed in an air bath. The temperature of the bath is changed from 21 °C to 24 °C in steps of about 1°C. Due to the high thermal inertia of the inductor the measurements are carried out 24 hours after each temperature change. DC resistance and inductance are measured as a function of temperature (figure 4a). The dc resistance value of the inductance corresponding to a temperature of 23 °C is taken as a reference. In figure 4b variation of inductance with resistance is plotted. Then, each inductance measurement is corrected by means of the dc measured resistance, taking into account the variation plotted in figure 4b.

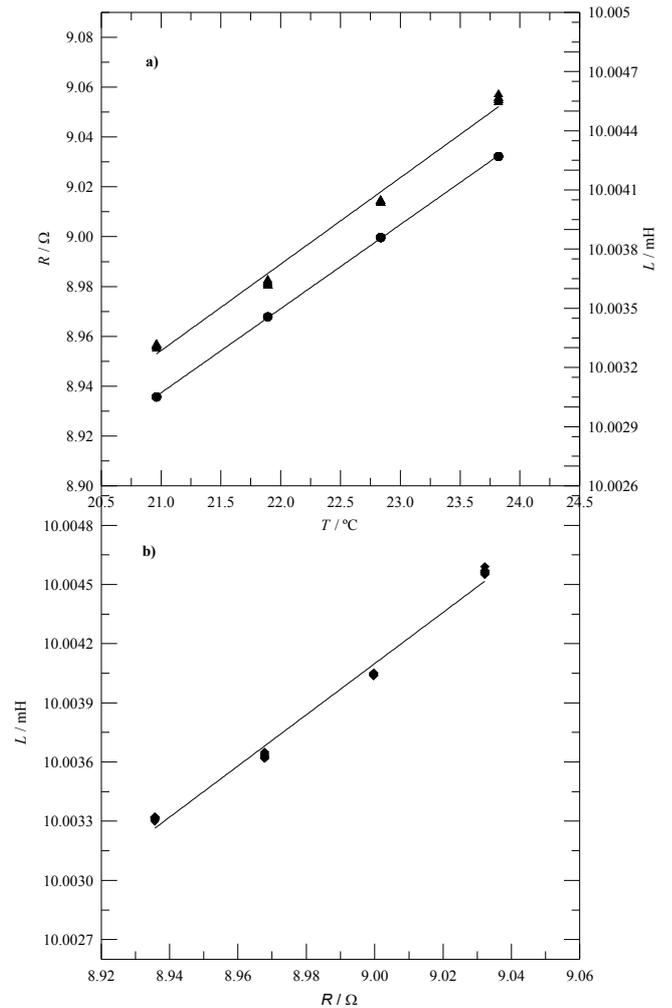


Fig. 4. a) Resistance (●) and inductance (▲) plotted as a function of temperature. b) Inductance vs resistance.

After applying the temperature correction, the main sources of uncertainty arise from the determination of capacitance, the value of two $1 \text{ k}\Omega$ fixed resistors R_1 and R_2 and the value of the small inductor L_{x0} .

By using the zero-substitution method a number of quantities in the model are negligible and do not contribute to uncertainty. Thus, the previously described Maxwell-Wien method ensures a relative expanded uncertainty of $30 \mu\text{H}/\text{H}$ for a 10 mH inductance standard at the frequency of 1 kHz.

Table 1. Uncertainty evaluation for the realization of a 10 mH inductance standard at 1 kHz.

Quantity	Type	Relative standard uncertainty in 10^{-6}
Repeatability (n=10)	A	6
Capacitance	B	6
AC resistance of the 1 k Ω resistor	B	4
AC resistance of the 1 k Ω resistor	B	4
Inductor L _{xo}	B	4
Temperature inductance	B	1.2
Temperature capacitance	B	1
Combined uncertainty		12
Expanded uncertainty		24

4. VALIDATION OF THE MEASUREMENT SYSTEM

Finally, validation of the measurement system previously described has been carried out by means of a 10 mH inductance standard recently calibrated at PTB. PTB certifies the 10 mH inductance with an uncertainty of 20 $\mu\text{H}/\text{H}$ ($k = 2$).

Differences obtained between values achieved with a Maxwell-Wien bridge at CEM and those certified by PTB are consistent with the 30 $\mu\text{H}/\text{H}$ ($k = 2$) CEM estimated uncertainty.

5. CONCLUSION

A Maxwell-Wien bridge has been designed for the realization of the inductance standard at CEM. It allows the obtaining of a 10 mH inductance at the frequency of 1 kHz, in terms of resistance and capacitance.

The different sources of uncertainty have been evaluated in detail and finally a relative expanded uncertainty of 30 $\mu\text{H}/\text{H}$ is reached. In the future, further efforts will be focused on the improvement of the system's accuracy and the development of a transformer ratio bridge specifically designed to scale inductance values from 100 μH up to 10 H.

REFERENCES

- [1] A. M. Thompson, D.G. Lampard, "A new theorem in electrostatics and its application to calculable standards of capacitance", *Nature*, 177, pp 88, 1956.
- [2] A. Campbell, "On a standard of mutual inductance", *Proceedings of the Royal Society*, 79, pp 428-435, 1907.
- [3] B. Hague, T.R. Ford, "Alternating Current Bridge Methods", 6th ed., London, Pitman, 1971.
- [4] B.P. Kibble, G.H. Rayner, "Coaxial AC Bridges", Bristol: Adam Hilger, 1984.
- [5] G. H. Rayner, B.P. Kibble, M.J. Swan, "On obtaining the henry from the farad", NPL Report DES(63).
- [6] R. Sedláček, L. Callegaro, F. Durbiano, "Resonance measurement of the inductance Q-factor in ultra acoustic frequency range utilizing impedance analyzer". XVII IMEKO World Congress Proceedings (Dubrovnik), pp. 650-651, 2003.
- [7] L. Callegaro, V. D'Elia, "EUROMET intercomparison of a 100 mH inductance standard", Draft A Report EUROMET EM-S.20, oct. 2005.
- [8] M. Côté, "A four-terminal coaxial pair Maxwell-Wien bridge for the measurement of self-inductance", *Conference On Precision Electromagnetic Measurements Digest*, Broomfield, pp 714-715, 2008.
- [9] Y. Gulmez, E. Turhan, G. Gulmez, H. Eckardt, P. Rather, "Intercomparison of a 10 mH inductance standard between UME-PTB", *Conference On Precision Electromagnetic Measurements Digest*, Washington, pp 498-499, 1998.
- [10] L. Callegaro, V. D'Elia, "Automated System for inductance realization traceable to ac resistance with a three-voltmeter method", *IEEE Trans, Instrum. Meas.*, 50, pp 1630-1633, 2001.
- [11] F. Overney, B. Jeanneret, "Realization of an inductance scale traceable to the quantum Hall effect using an automated synchronous sampling system", *Metrologia*, 47, pp 690-698, 2010.
- [12] I. Robinson, J. Bellis, "EUROMET intercomparison of AC Voltage Ratio", Draft A Report, march 2011.
- [13] Y. A. Sanmamed, J. Díaz de Aguilar, A. Sánchez and M. Neira, "Patrón Nacional de Inductancia", IV Congreso Español de Metrología, S-O-T-9-1, 2009.
- [14] B. N. Taylor, T. J. Witt, "New international electric reference standards based on the Josephson and quantum Hall effects", *Metrologia*, 26, pp 47-62, 1989.
- [15] T. L. Zapf, "Calibration of inductance standard in the Maxwell-Wien bridge circuit", *J. Res. NBS-C*, 65C, pp 183-188, 1961.
- [16] International Standard Organization, *Guide to the Expression of Uncertainty in Measurement*. Geneva, Switzerland: ISO, 1991.