# APPLICATION OF FEEDBACK LINEARIZATION IN THERMORESISTIVE SENSORS CHARACTERIZATION AND FEEDBACK MEASUREMENT SYSTEMS

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**Abstract:** This paper presents the application of feedback linearization for the characterization of thermoresistive sensors and for feedback measurement systems using these sensors kept at constant temperature. This technique allows the generation of an input signal which is proportional to the electric power of the sensor, simplifies the characterization with the possibility of determining thermal conductance and intrinsic time constant of the sensor in a single test. Concerning the feedback measurement system, this technique improves features of the measurement system with the possibility of use a simple linear controller with good performance in the operation point. Experimental results of characterization and measurement are presented.

**Key words:** thermoresistive sensor, closed loop system, characterization of sensor, feedback linearization, virtual instruments.

## 1. INTRODUCTION

This paper presents an application of feedback linearization for the characterization of thermoresistive sensors and feedback measurement systems, with the aid of an experimental platform (in this case, data acquisition system with LabVIEW).

The main objective of thermoresistive sensors characterization is determining the parameters of the equations which govern the static and dynamic behavior of the sensors. Some works have already been developed in order to determine these parameters through electric signals and radiation steps. In [1], the dynamic response of sensor is measured by electric excitation steps and radiation step, in [2], two steps of current are applied to determine two different apparent time constants and the dynamic response is obtained from these values. In [3], an algorithm is proposed to determine the parameters of the sensor using the curve I x V and, in [4], a method is presented using sinusoidal signals for extracting the parameters experimentally. These works relate to the dynamic characterization of sensor, using different ways to determine the parameters, with the static characterization made in separated test. The feedback linearization allows application of staircase signal proportional to electric power of sensor, making the determination of time constant and thermal conductance of sensor possible in a single test [5]. The system order (number of time constants) can be checked more easily, since the feedback linearization allows the use of an input variable (excitation) related linearly with the temperature of sensor.

The thermoresistive sensors can be used to measure temperature, incident radiation, or fluid velocity. The following methods can be used for these measurements: constant voltage, constant current and constant temperature [5] [6]. The measurement system that will be used in this work keeps the sensor temperature constant providing a shorter time constant compared to the others.

In constant temperature measuring method, the sensor is part of a closed-loop system, and energy variations caused by the quantity that intend to be measured are compensated by system, from change the electric energy (voltage or current) on the sensor. In conventional systems are used Wheatstone's bridges with the sensor in one arm, which is supplied by the output voltage of the differential amplifier [7].

In [8], a system where the sensor is powered by a current signal with pulse width modulation was proposed. The behavior and performance of both systems depend on the operating point, due to nonlinearity.

Thus, the idea is to use the technique of feedback linearization to provide best performance of control system at various points of operation, where the control variable, that is also the measurement variable, is proportional to electric power of sensor.

## 2. OBJECTIVE

This paper aims to contribute in thermoresistive sensors characterization using feedback linearization technique and its application in feedback measurement systems. It will simplify the characterization, allowing validation of the sensor model and improving performance of feedback measurement system.

## 3. THE THERMORESISTIVE SENSOR

Two equations are used to describe thermoresistive sensors behavior. The first equation represents energy balance and is given by:

$$\alpha.S.H + P_S(t) = G_{th}.[T_S(t) - T_a(t)] + C_{th}\frac{dT_S(t)}{dt} \quad (1)$$

where $\alpha SH$ represents the radiation absorbed by the sensor per unit of time; $\alpha$ is the coefficient of transmissivity-absorptivity of the sensor; $S$ is the surface area of the sensor; $H$ is the incident radiation; $P_S$ is the electric power of sensor; $G_{th}.(T_S-T_a)$ is the energy lost by the sensor to the environment per unit of time; $C_{th}.(dT_S / dt)$ is the change in internal energy of sensor per unit of time; $G_{th}$ is the thermal conductance; $C_{th}$ is the thermal capacitance; $T_S$ is the temperature of the sensor; $T_a$ is the ambient temperature. This equation is valid for a sensor without substrate or encapsulation.

The second equation represents the relation between sensor resistance and temperature. The thermoresistive sensor used in this research is a NTC (Negative Temperature Coefficient). In this case, the relation can be given by:

$$R_S = A \cdot e^{\frac{B}{T_S}} \quad (2)$$

$$A = R_0 \cdot e^{-\frac{B}{T_0}} \quad (3)$$

where $A$ is the sensor resistance when the temperature tends to infinity; $B$ is the characteristic temperature of the sensor; $R_0$ is the sensor resistance to a reference temperature; $T_0$ is the reference temperature of the sensor. Other equations that describe the thermoresistive sensor behavior can be found in [6].

Usually the thermoresistive sensors characterization is done by combination of three experimental tests:

In the first experimental test are made measurement of electrical resistance in an environment with adjustment of temperature. The parameters $A$ and $B$ can be determined from the equation (2) taking two pairs of values ($R_S$,$T_S$). One can also use a method of curve fitting for various pairs of values ($R_S$,$T_S$). In this experimental test the heating by thermal radiation and self heating by Joule effect must be zero or very small.

In the second experimental test, the sensor is immersed in the fluid where one wants to determine $G_{th}$. The temperature of this medium is kept constant and the sensor is heated by Joule effect by a constant electric current with values $I_i$ (i=1 to n). In steady state ($dT_S/dt$=0) the value of $G_{th}$ can be determined by curve fitting with the values of $P_S$ and ($T_S$-$T_a$).

In the third experimental test, is measured the electrical resistance of the sensor (temperature) during a time interval. The time constant of the sensor is determined by curve fitting, or from the falling or rising time of the temperature [9].

Experimentally is not possible to directly apply an excitation signal in form of electric power. It is common to work with a voltage signal or electric current to heat the sensor. Using electric current:

$$P_S(t) = R_S(T_S(t)) \cdot I_S^2(t) \quad (4)$$

Substituting (4) in (1):

$$\alpha SH + R_S(T_S(t)) I_S^2(t) = G_{th}[T_S(t) - T_a(t)] + C_{th}\frac{dT_S(t)}{dt} \quad (5)$$

The thermoresistive sensor characterization with electric current causes difficulties due the nonlinear differential equation (5). This happens because in non linear systems the time constant varies, when working at different operation points. This time constant is called apparent time constant and depends on the electric current applied to the sensor [2].

The feedback linearization allows determine the intrinsic time constant of the sensor in the second experimental test previously described. This is possible because with this technique one can work with a linearized system.

## 4. FEEDBACK LINEARIZATION

The system used in feedback linearization is shown in Figure 1.

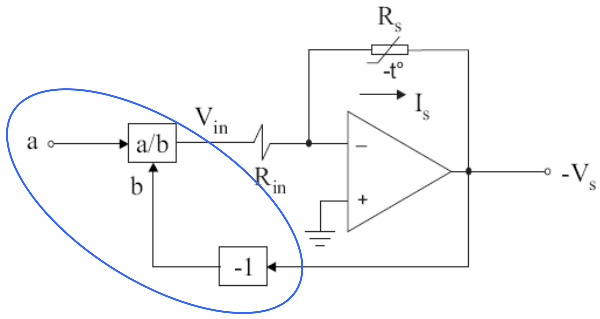


**Fig 1. Basic electronic system with feedback linearization**

For this system, one can deduce:

$$V_{in}(t) = R_{in}(t).I_S(t) \quad (6)$$

$$V_{in}(t) = \frac{a(t)}{b(t)} = \frac{a(t)}{V_S(t)} \quad (7)$$

$$a(t) = R_{in}.V_S(t).I_S(t) \quad (8)$$

By analyzing those equations, one can see that the input value *a(t)* is equal to the product between the power of the sensor and the input resistance ($R_{in}$). *a(t)* is a virtual variable generated in LabVIEW (encircled region in Figure 1).

By substituting the value of *a(t)* in (1) and considering the incident radiation equal to zero ($H$ = $0$), (1) can be rewritten as:

$$a(t) = R_{in}.G_{th}.[T_S(t) - T_a(t)] + R_{in}.C_{th}.\frac{dT_S(t)}{dt} \quad (9)$$

One can observe by (9) that the system with feedback linearization is linear which respect to the new input variable *a(t)*, which is an independent variable of sensor voltage and current, unlike the electric power of sensor $P_S$

defined in (1), which is obtained from $I_S(t)$ (generated by $a(t)$) and $V_S(t)$.

For $T_S(t)$ initially equal to $T_a$, and $T_a$ constant, the step response of $T_S(t)$ for an $a(t)$ step with amplitude $A_a$ is given by:

$$T_S(t) - T_a = A_a . R_{in}^{-1} . G_{th}^{-1} \left( 1 - e^{-\frac{t}{\tau}} \right) \quad (10)$$

In this equation $\tau$ is the intrinsic time constant of the sensor given by $C_{th}/G_{th}$.

## 5. EXPERIMENTAL PLATFORM

The experimental platform is shown in Figure 2.

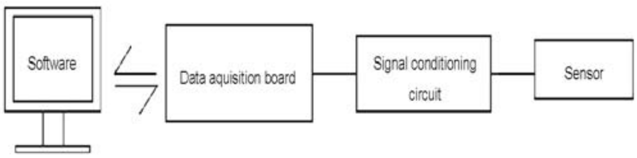


**Fig 2. Experimental platform**

It is composed of: a closed environment inside of which the sensor is placed. In this environment the incident radiation is equal to zero ($H$=0) and the ambient temperature is monitored by a thermometer;

A signal conditioning circuit (Figure 3) responsible by adjusts in voltage levels of DAC and ADC converters, of data acquisition board;

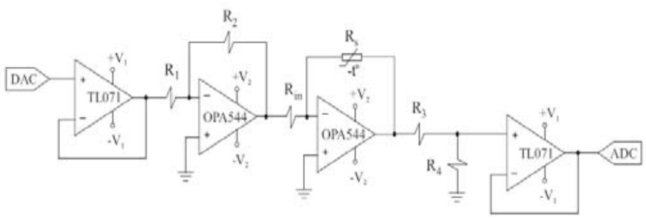


**Fig. 3. Signal conditioning circuit**

A data acquisition board PCI6024E model, produced by National Instruments and a computer with LabVIEW software.

## 6. MEASUREMENT SYSTEM

The Figure 4 shows the block diagram of the measurement system.

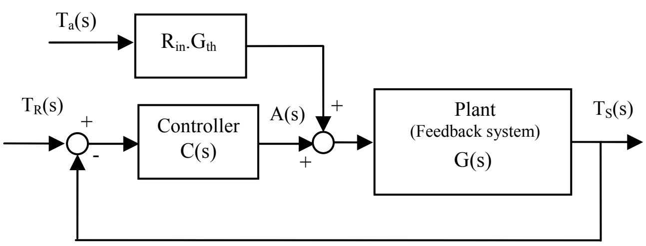


**Fig 4. Block diagram of control structure**

The input variable of the system with feedback linearization (Plant) is $A(s)$, which is also the measurement variable. $T_S(s)$ is the observed variable (temperature of the sensor), which must be kept constant by the control system.

Similar work was developed in [8] that used a PWM signal to obtain a signal proportional to the square of sensor current. In this case, the system performance depends on the operating point.

The linear relation between sensor temperature and the measurement variable, due to feedback linearization, improves the control performance and the measurement process.

It was used a controller PI to obtain a time constant in closed loop less than the time constant of the sensor, through cancellation of pole and zero.

$$C(s) = K_P \cdot \left( 1 + \frac{K_I}{s} \right) \quad (11)$$

$$C(s) \cdot G(s) = K_P \left( 1 + \frac{K_I}{s} \right) \cdot \frac{\left( R_{in} . G_{th} \right)^{-1}}{\tau . s + 1} \quad (12)$$

By choosing,

$$K_P = \frac{R_{in} \cdot G_{th} \cdot \tau}{\tau_f} \quad (13)$$

$$K_I = \frac{1}{\tau} \quad (14)$$

The transfer function in closed loop is:

$$\frac{T_S(s)}{T_R(s)} = \frac{1}{\tau_f s + 1} \quad (15)$$

In this equation, $\tau_f$ is the time constant of the closed loop pole.

The ambient temperature in Figure 4 is considered as a disturbance to the control system. The energy variation caused by this disturbance will be compensated by $A(s)$ to keep constant the temperature of the sensor.

By analyzing the transfer function in closed loop of ambient temperature (measurand) to $A(s)$:

$$\frac{A(s)}{T_a(s)} = -\frac{R_{in} \cdot G_{th}}{\tau_f s + 1} \quad (16)$$

One can observe that the influence of disturbance is related to the time constant of closed loop designed.

Similar function is obtained for $A(s)/H(s)$, where $H$ is the thermal radiation incident (1) and (5).

The ambient temperature can be determined by (9) in steady state (temperature of the sensor kept constant by control system $dTs/dt$=0) by:

$$\overline{T_a} = \overline{T_S} - \frac{\overline{a}}{R_{in} G_{th}} \quad (17)$$

## 7. RESULTS

For a bead-bare sensor (without packaging or substrate) the manufacturer furnishes a table with the values of temperature of the sensor by the electrical resistance. Two different temperature have been selected and the values of *A* and *B* were calculated by equation (2): $A=0{,}01354\Omega$ and $B=3342{,}2K$.

The response to signal *a(t)* as a rising staircase is illustrated in Figure 5 and it corresponds to sensor temperature response, in time domain, given by (10).

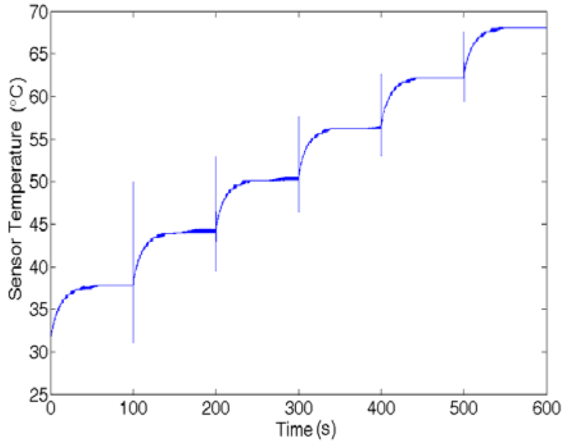


**Fig 5. Staircase response to signal *a(t)***

According to the response, the intrinsic time constant of the sensor was calculated through the curve fitting by means of least square method. The obtained data are shown in Table 1, with the mean value of intrinsic time constant $\tau$ = 10.3 s.

**Table 1. Power of sensor and intrinsic time constant**

| Step | a (W.Ω) | Intrinsic time constant (s) |
|---|---|---|
| 1 | 10 | 10,5 |
| 2 | 15 | 10,6 |
| 3 | 20 | 10,2 |
| 4 | 25 | 10,2 |
| 5 | 30 | 9,8 |
| 6 | 35 | 10,4 |

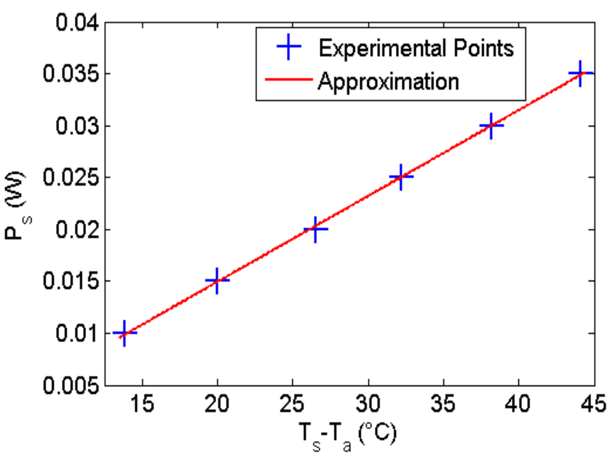


**Fig 6. $P_S$ x $T_S$-$T_a$ after transient, at each step of Figure 5**

The graph shown in Figure 6 was constructed based on (9) for power values of sensor ($P_S$) and temperature variations ($T_S$-$T_a$), in the steady state ($dT_S/dt$=0) for each step in Figure 5. The slope of straight line that better fits experimental points was determined by means of least squares method. This slope is the thermal conductance, which for the used sensor was 0.8267 mW/°C.

For the sensor with encapsulation used, was determined the relation between sensor resistance and temperature of the sensor (Figure 7) given by (2), in an environment with temperature control. The curve fitting was used and the values of *A* and *B* was calculated: $A=\ 0{,}00271\Omega$ and $B=4035K$.

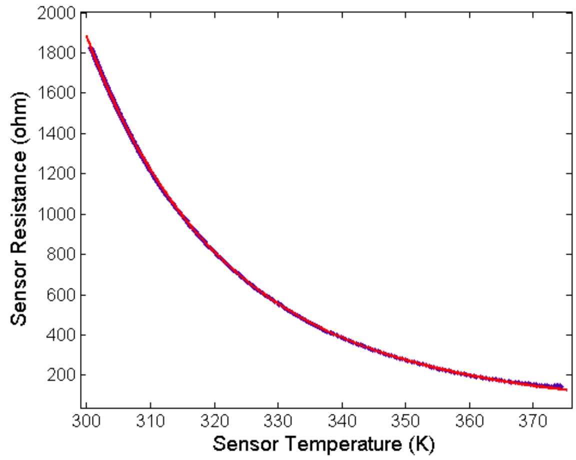


**Fig. 7. Characteristic $R_S$ x $T_S$, sensor with encapsulation**

The Figure 8 illustrates the response to the signal *a(t)* as a rising staircase for a sensor with encapsulation.

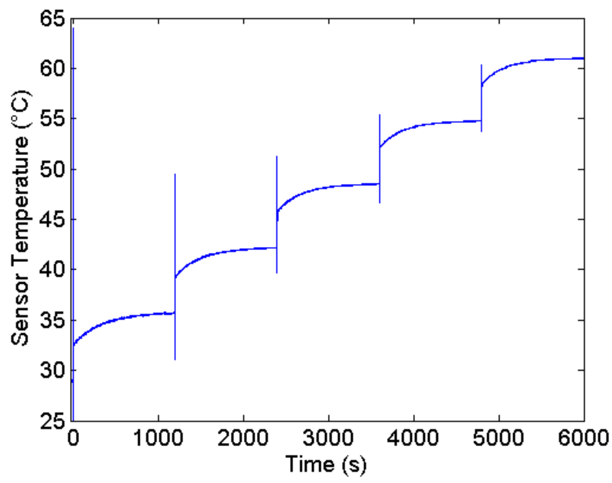


**Fig 8. Staircase response to signal *a(t)* for a sensor with encapsulation**

The function that best approximates the response was a function of second order. The obtained data are shown in Table 2, with the mean value of time constants $\tau_1$= 1,8 s and $\tau_2$= 279,1 s.

**Table 2. Power of the sensor and time constant (sensor with encapsulation)**

| Step | a (W.Ω) | Time constant (s) | |
|---|---|---|---|
| | | $\tau_1$ | $\tau_2$ |
| 1 | 40 | 2,2 | 303,6 |
| 2 | 70 | 1,8 | 286,4 |
| 3 | 100 | 1,7 | 275,1 |
| 4 | 130 | 1,8 | 267,1 |
| 5 | 170 | 1,7 | 263,4 |

The slope of straight line (Figure 9) that better fits experimental points was determined and the total thermal conductance was 4.903 mW/°C.

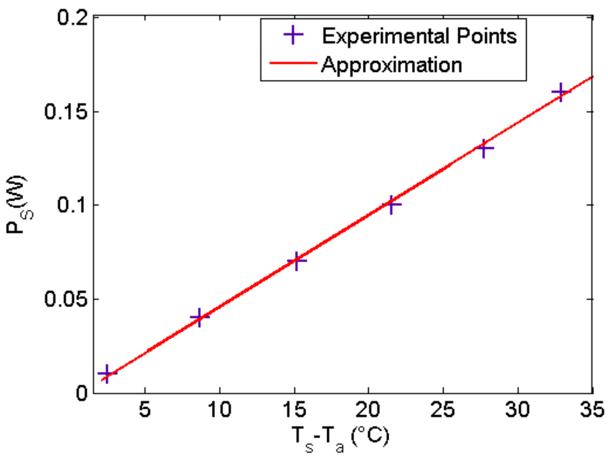


**Fig 9.** $P_S$ **x** $T_S$**-**$T_a$ **after transient, at each step of Figure 8**

For the measurement system, was used an oven with temperature control and a thermometer Kiltler-100TC with serial interface, to measure the temperature inside the oven. The relation between the temperature measured by the feedback system, using (17) and the temperature read by the thermometer can be seen in Figure 10.

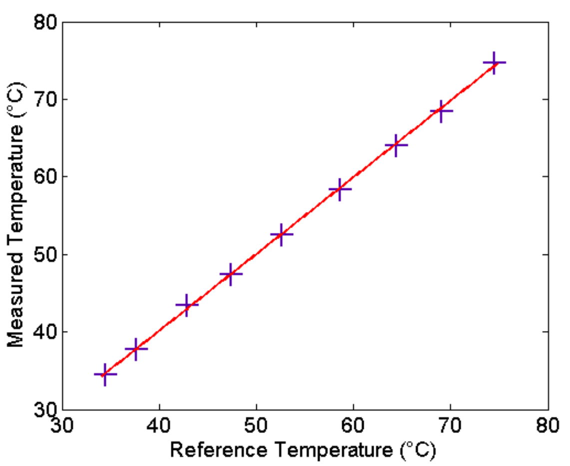


**Fig 10. Measured Temperature x Reference Temperature**

The oven was placed in nine different operating points. The average temperature for each operation point was measured during a period of 10 seconds with the feedback system and with the thermometer. The slope of straight line that better fits the measured points was 0,98.

The measured values at each operating point are shown in Table 3, with the error in each measure.

**Table 3. Ambient temperature measured**

| Reference Temperature (°C) | Measured Temperature (°C) | Error (°C) |
|---|---|---|
| 34,5 | 34,4 | 0,1 |
| 37,7 | 37,8 | -0,1 |
| 42,9 | 43,4 | -0,5 |
| 47,4 | 47,4 | - |
| 52,6 | 52,6 | - |
| 58,6 | 58,3 | 0,3 |
| 64,4 | 64,0 | 0,4 |
| 69,0 | 68,5 | 0,5 |
| 74,5 | 74,7 | -0,2 |

## 8. CONCLUSION

In this work, the use of feedback linearization applied to thermoresistive sensors characterization and measurement systems was presented. The technique allows the generation of a signal proportional to the electric power of the sensor, which has linear relation with the temperature of the sensor. For sensors without encapsulation, this simplifies the sensor characterization methodology (determination of $G_{th}$ and time constant in a single test) and improves features of the measurement system (performance of control and the measurement process).

The feedback linearization was also applied in characterization of sensors with encapsulation, and allowed to identify two time constants.

It was shown that the system can be used to measurement of temperature, although not the most suitable compared to existing systems for consumption issues and the result depends of thermal conductance. As perspectives for future works is intended to study the technique with the feedback system for measuring of thermal radiation.

Using the feedback system with controller PI, one can adjust the time constant of the response to variation of measurand (thermal radiation or temperature) for one value less than the intrinsic time constant of the sensor, and adjust to the time response adequate to application.

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