ROBOTIC MANIPULATORS FOR ATTITUDE SENSORS CALIBRATION: THE MEASUREMENT MODEL AND UNCERTAINTY CALCULATION

Bruno Cabral de Oliveira Dutra [1], *Guilherme Augusto Silva Pereira* [2], *Flávio Henrique de Vasconcelos* [3]

Departamento de Engenharia Elétrica, Universidade Federal de Minas Gerais, Belo Horizonte, MG, Brazil.
[1] brunocodutra@gmail.com, [2] gpereira@ufmg.br, [3] fvasc@dee.ufmg.br

Abstract: Recent works have proposed the use of industrial robots as a standard for intermediary calibration of inertial sensors such as accelerometers and magnetometers. In this paper it is developed the (mathematical) measurement model to be used for the determination of the measurement result and the uncertainty in multiple links robot manipulators. This model relates many random variables, such as the linear and angular positions of the robot's end-effector, and correction factors for the determination of both the mean and the measurement uncertainty for a simple three link manipulator case using both the law of uncertainty propagation (LDU) technique and the Monte-Carlo method and the results are compared.

Key words: measurement model, calibration, inertial sensors, robotic manipulators, attitude and heading reference system.

1. INTRODUCTION

Attitude and Heading Reference Systems (AHRS) or its simplified version called Inertial Measurement Units (IMU), are attitude (and Heading) estimation equipments, generally constituted by sensors such as accelerometers, gyroscopes, and magnetometers and a embedded software which process the data from the sensors. The most common applications for these systems are in the field of aerospace engineering but it is extensively used to stabilize other plants, such as offshore oil extraction platforms [1].

Sensor calibration is one of the first steps in the process of building an AHRS. Equipment and instrumentation to calibrate such sensors are commercially available in the market, but it is generally costly prohibitive for some research laboratories and small industries [2]. In order to reduce these costs, recent scientific works have been proposed the use of industrial robots to substitute the commercial devices [3, 4]. The main idea on these works is to use the robot manipulator as a calibrated [10] standard and, at the same time, as an actuator, able to locate the sensors in different angular positions.

In this context, this paper presents an analytical measurement model for a multiple degree-of-freedom manipulators. The Generalized Law of Propagation of Uncertainties (GLPU) [5, 9] is the method employed for calculating the combined uncertainty which relates the contributions of the end-effector linear, angular positions, correction factors, etc. In this paper, the result obtained with GLPU is compared with that obtained by the Monte-Carlo method for a simple 3-degree-of-freedom manipulator.

2. OBJECTIVES

In order to calibrate accelerometers and magnetometers in IMUs, the work by Renk et al [3] have proposed the use of an industrial manipulator. Basically, the proposed model for the k_{ism} accelerometer is (a similar equation is found for a magnetometer):

$$V_k = |\vec{g}| S_k \left[R_{\hat{k}}(\psi) R_{\hat{j}}(\phi)\hat{g}\right]^T \left[{}^{e}_{b}R\, R_{\hat{k}}(\gamma_k) R_{\hat{j}}(\beta_k)\hat{\imath}_e\right] + \delta_k \quad (1)$$

where V_k is the sensor output in volts, g is the acceleration vector, and S_k and δ_k are the sensor sensibility and offset respectively. The first bracket represents the orientation of robot base, which depend on the angles ψ and ϕ, and the second bracket represents the orientation of the sensor in relation to the robot base. This orientation depends on the two angles, γ_k and β_k, but also on the rotation matrix ${}^{e}_{b}R$, which is the matrix that represents the end-effector rotation in relation to the robot base and depends on robots variables, such as the angles of each joint. For sensor calibration, the model proposed in [3] depends on variables such as S_k and δ_k, which are in fact the parameters of interest, and also on others such as ψ, ϕ, γ_k and β_k. In Equation (1), it is assumed that g and ${}^{e}_{b}R$ are exactly known.

Although ${}^{e}_{b}R$ may be computed using the parameters of each of the robot's links, namely the length and the torsion, and also the joint variables, angle and/or offset [5], from the metrological point of view, it is necessary to understand how the inaccuracy of these parameters propagate to ${}^{e}_{b}R$ and, consequently, how it influence the estimation of the parameters of interest S_k and δ_k in Equation (1).

Thus, the main goal of this paper is to present a mathematical measurement model that enables the computation of ${}^{e}_{b}R$. The approach adopted in this paper is based on a linearization of the robot's kinematic model.

3. METHODOLOGY

Using the Denavit-Hartemberg (DH) methodology [6] it is usually quite simple to determine the direct kinematic model of a manipulator. This model is usually written as a transformation matrix, T, that contains both a rotation and a position matrix that represent the orientation and the position of the manipulator end-effector in relation to its base.

To understand how robot transformation matrix is computed, firs consider a transformation ${}^{i-1}_{i}T$, between the frames *i* and *i-1* attached on two consecutive links:

$$\begin{bmatrix} {}^{i-1}\mathbf{r} \\ 1 \end{bmatrix} = {}^{i-1}_{i}\mathbf{T}\left({}^{i-1}_{i}\mathbf{p}\right) \cdot \begin{bmatrix} {}^{i}\mathbf{r} \\ 1 \end{bmatrix},$$

where ${}^{i}r$ e ${}^{i-1}r$ are the spatial vector positions and ${}^{i-1}_{i}p = [\theta_i, L_i, \alpha_{i-1}, a_{i-1}]$ are the angles and the linear dimensions of the manipulator, which are also called Denavit-Hartemberg (DH) parameters. Following the modified DH convention, ${}^{i-1}_{i}T$ is written as:

$${}^{i-1}_{i}\mathbf{T} = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & a_{i-1} \\ \sin(\theta_i)\cos(\alpha_{i-1}) & \cos(\theta_i)\cos(\alpha_{i-1}) & -\sin(\alpha_{i-1}) & -d_i\sin(\alpha_{i-1}) \\ \sin(\theta_i)\sin(\alpha_{i-1}) & \cos(\theta_i)\sin(\alpha_{i-1}) & \cos(\alpha_{i-1}) & d_i\cos(\alpha_{i-1}) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Assuming that is it possible to find transformation between two consecutive robot links, is it possible to obtain the global robot transformation matrix as a composition of N matrix as:

$${}^{0}_{N}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot \ldots \cdot {}^{N-1}_{N}T \qquad (2)$$

However, the transformation matrix itself is not sufficient. It is usual the need for the position and rotation of a given frame in relation to another, as is the case of the calibration problem that motivated this work. Fortunately, this information is directly obtained from the transformation matrix.

Assume a transformation ${}^{B}_{A}T$ from frame A to B is given. Call by $Org\left({}^{B}_{A}T\right): M(4,4) \mapsto \Re^3$ the function that returns the position of the origin of A in relation to B and by $Rot\left({}^{B}_{A}T\right): M(4,4) \mapsto \Re^3$ the function that returns the rotation of A in relation to B in the form of rotation vectors. These functions are given by:

$$Org\left({}^{B}_{A}\mathbf{T}\right) = \begin{bmatrix} {}^{B}_{A}t_{14} \\ {}^{B}_{A}t_{24} \\ {}^{B}_{A}t_{34} \end{bmatrix}$$

$$Rot\left({}^{B}_{A}\mathbf{T}\right) = \frac{\theta}{2\sin(\theta)} \begin{bmatrix} {}^{B}_{A}t_{32} - {}^{B}_{A}t_{23} \\ {}^{B}_{A}t_{13} - {}^{B}_{A}t_{31} \\ {}^{B}_{A}t_{21} - {}^{B}_{A}t_{12} \end{bmatrix}, \qquad (3)$$

where ${}^{B}_{A}t_{ij}$ represents the element in line i and column j of ${}^{B}_{A}T$ and

$$\theta = arc\ cos\left(\frac{{}^{B}_{A}t_{11} + {}^{B}_{A}t_{22} + {}^{B}_{A}t_{33} - 1}{2}\right)$$

An important detail is that function *Rot* is undefined when θ is *0* or π.

It can be noticed that, in the case of a robot, the functions in Equation (3) only depends on the DH parameters of the manipulator. Therefore, it is clear that the uncertainty on the estimation of end-effector position and orientation only depends on the uncertainty related to the DH parameters. In what follows we mathematically write equations for this dependency.

Consider two frames A and B related by a transformation matrix ${}^{B}_{A}T$. Let ${}^{B}org_A$ and ${}^{B}rot_A$ be the origin position and rotation of A in relation to B, respectively, and $Q_{{}^{B}org_A}$ and $Q_{{}^{B}rot_A}$ be the covariances matrix i.e., the uncertainty matrix, that represent the uncertainty associated with these vectors. Also, assume that the uncertainties associated with the robot's DH parameters are known an represented by $Q_{{}^{B}_{A}P}$. Finally, call $J_{Org({}^{B}_{A}T)}$ and $J_{Rot({}^{B}_{A}T)}$ the Jacobian of functions *Org* and *Rot* in relation to ${}^{B}_{A}P$. The Generalized Laws of Propagation of Uncertainties (GLPU) [5] is then given by:

$$\begin{aligned} \mathbf{Q}_{{}^{B}\mathbf{org}_A} &= \mathbf{J}_{Org({}^{B}_{A}\mathbf{T})} \cdot \mathbf{Q}_{{}^{B}_{A}\mathbf{p}} \cdot \mathbf{J}^T_{Org({}^{B}_{A}\mathbf{T})} \\ \mathbf{Q}_{{}^{B}\mathbf{rot}_A} &= \mathbf{J}_{Rot({}^{B}_{A}\mathbf{T})} \cdot \mathbf{Q}_{{}^{B}_{A}\mathbf{p}} \cdot \mathbf{J}^T_{Rot({}^{B}_{A}\mathbf{T})}. \end{aligned} \qquad (4)$$

It should be noted that the $Q^{B}_{A\,p}$ matrix is the input quantities uncertainty matrix and that $Q_{B\,org_A}$ *and* $Q_{B\,rot_A}$ are the output quantities matrix. Likewise, $J_{rot({}^{B}_{A}\boldsymbol{T})}$ is sensitivity matrix [5].

In some particular cases the manipulator parameters can be considered independent, and $Q_{{}^{B}_{A}P}$ generally being a diagonal and each element can be obtained using standard PDF identification techniques [7].

4. EXAMPLE

The aforementioned propagation method will me exemplified in a three degree-of-freedom (DOF) planar manipulator shown in Figure 1. For this robot we assume the following DH parameters:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2

3	0	L_2	0	θ_3

The first step of the methodology is to find the robot transformation matrix by the composition of several consecutives link transformation matrices as in Equation (2). For the three DOF robot we have three transformation matrices:

$$ {}^{0}_{1}\mathbf{T} = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}\mathbf{T} = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}\mathbf{T} = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

By using such matrices and Equation (2) we have:

$$ {}^{0}_{3}\mathbf{T} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 & L_1\cos(\theta_1) + L_2\cos(\theta_1+\theta_2) \\ \sin(\phi) & \cos(\phi) & 0 & L_1\sin(\theta_1) + L_2\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (5) $$

where $\phi = \theta_1 + \theta_2 + \theta_3$.

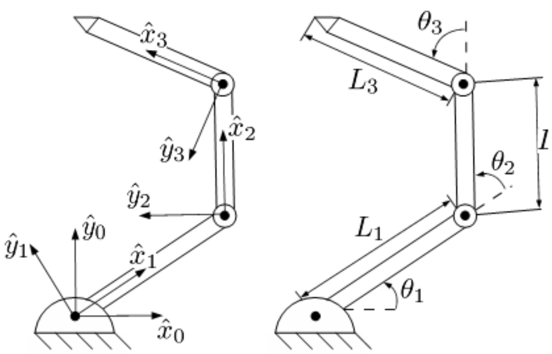

Figure 1. Manipulator used in the simulations.

In this example, for the sake of simplicity, it will be assumed that the manipulator dimensions, represented by L_1 and L_2, are known with absolute precision, meaning that the only uncertainties are associated to θ_1, θ_2 and θ_3, which will be represented, respectively, by $\delta^2_{\theta_1}$, $\delta^2_{\theta_2}$ and $\delta^2_{\theta_3}$. In this way:

$$ {}^{0}_{3}\mathbf{p} = [\theta_1, \theta_2, \theta_3]^T $$

$$ \mathbf{Q}_{{}^{0}_{3}\mathbf{p}} = \begin{bmatrix} \sigma^2_{\theta_1} & 0 & 0 \\ 0 & \sigma^2_{\theta_2} & 0 \\ 0 & 0 & \sigma^2_{\theta_3} \end{bmatrix} $$

From the forward kinematic model of the robot (Equation (5)) we have:

$$ {}^{B}\mathbf{org}_A = Org\left({}^{0}_{3}\mathbf{T}\right) = \begin{bmatrix} L_1\cos(\theta_1) + L_2\cos(\theta_1+\theta_2) \\ L_1\sin(\theta_1) + L_2\sin(\theta_1+\theta_2) \\ 0 \end{bmatrix} $$

$$ {}^{B}\mathbf{rot}_A = Rot\left({}^{0}_{3}\mathbf{T}\right) = \begin{bmatrix} 0 \\ 0 \\ \theta_1+\theta_2+\theta_3 \end{bmatrix} $$

The Jacobians of such functions are then given by:

$$ \mathbf{J}_{Org({}^{0}_{3}\mathbf{T})} = \begin{bmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -L_1\sin(\theta_1) - L_2\sin(\theta_1+\theta_2) & -L_2\sin(\theta_1+\theta_2) \\ L_1\cos(\theta_1) + L_2\cos(\theta_1+\theta_2) & L_2\cos(\theta_1+\theta_2) \\ 0 & 0 \end{bmatrix} $$

$$ \mathbf{J}_{Rot({}^{0}_{3}\mathbf{T})} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} $$

Finally, by applying Equation (4) we find the uncertainties related to rotation and translation as:

$$ \mathbf{Q}_{{}^{B}\mathbf{org}_A} = \begin{bmatrix} \sigma^2_{\theta_1}A^2 + \sigma^2_{\theta_2}B^2 & \sigma^2_{\theta_1}AC + \sigma^2_{\theta_2}BD & 0 \\ \sigma^2_{\theta_1}AC + \sigma^2_{\theta_2}BD & \sigma^2_{\theta_1}C^2 + \sigma^2_{\theta_2}D^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} $$

$$ \mathbf{Q}_{{}^{B}\mathbf{rot}_A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\theta_1} + \sigma^2_{\theta_2} + \sigma^2_{\theta_3} \end{bmatrix} $$

6. SIMULATIONS

Using a Matlab simulator [8], we have implemented the aforementioned propagation method in a three degree-of-freedom planar manipulator. In our simulations, it was assumed that our parameters were exactly know, except for the variables θ_i that represent angles measured by sensors such as rotary encoders or potentiometers. For these sensors it was assumed a normal PDF with zero mean and know standard deviation. Therefore, the covariance matrix of the parameters was represented by a 3x3 diagonal matrix. We arbitrarily choose numerical values for these quantities as as:

$$ L_1 = 0.5\,, \quad L_2 = 1.0 $$

$$ {}^{0}_{3}\mathbf{p} = [3.8581 \;\; 1.9291 \;\; 5.7872]^T $$

$$ \mathbf{Q}_{{}^{0}_{3}\mathbf{p}} = \begin{bmatrix} 0.0004 & 0.0000 & 0.0000 \\ 0.0000 & 0.0004 & 0.0000 \\ 0.0000 & 0.0000 & 0.0004 \end{bmatrix} $$

These parameters were applied in Equation (4) yielding the following uncertainty matrices:

$$ \mathbf{Q}_{{}^{B}\mathbf{org}_A} = \begin{bmatrix} 0.3493 \cdot 10^{-3} & 0.3291 \cdot 10^{-3} & 0.0000 \\ 0.3291 \cdot 10^{-3} & 0.4104 \cdot 10^{-3} & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix} $$

$$ \mathbf{Q}_{{}^{B}\mathbf{rot}_A} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0012 \end{bmatrix} $$

These results were compared with results obtained using Monte-Carlo simulation. In this simulation 5000 tuples of input angles were used yelding the following uncertainty matrices:

$$ \mathbf{Q}_{{}^{B}\mathbf{org}_A} = \begin{bmatrix} 0.6955 \cdot 10^{-3} & 0.6164 \cdot 10^{-3} & 0.0000 \\ 0.6164 \cdot 10^{-3} & 0.6420 \cdot 10^{-3} & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix} $$

$$ \mathbf{Q}_{{}^{B}\mathbf{rot}_A} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0012 \end{bmatrix} $$

Notice that, for this specific robot, since the equation for orientation is linear, the analytical results where identical to the ones obtained via Monte-Carlo simulation. For translation, since a numerical comparison is hard to be done, we plot the results in graph to facilitate the analysis of the results. Figure 2 shows, as black-dots the Monte-Carlo samples that represent the image of the function represented by the forward kinematics of the robot. In the same figure we have plotted in blue and red, respectively, the ellipses that represent the uncertainty obtained using Monte-Carlo and the method presented before. Each ellipse represents the region of three standard-deviations, i.e. the region where is expected to find approximately 99% of the samples.

By Figure 2, it is possible to see that, although the two covariance matrix seems to be different, the region represented by the ellipses is quite similar. The difference observed may be explained by non-linearity of the robot's forward kinematic equation. Therefore, a different robot could present different results.

5. CONCLUSIONS

In this paper we derive the (mathematical) measurement model that can be used for the determination of the orientation estimations of the end-effector of a industrial manipulator robot. This is important if the manipulator is used a secondary calibration standard for orientation sensors and inertial measurement units. We have compared the analytical results obtained using a Generalized Law of Propagation of Uncertainties (GLPU) presented in the paper with the ones obtained via Monte-Carlo for a fairly simple manipulator. Even in this simple case, the results obtained in the final PDF are not identical, due to the non-linearities on the robot model. Therefore, the use of one method or the other will have to be chosen for each robot been used. Moreover, it should be noted that there are several other correction factors derived from sensor data-sheet, books, etc. that could be added to this model making more complete, but also more complex.

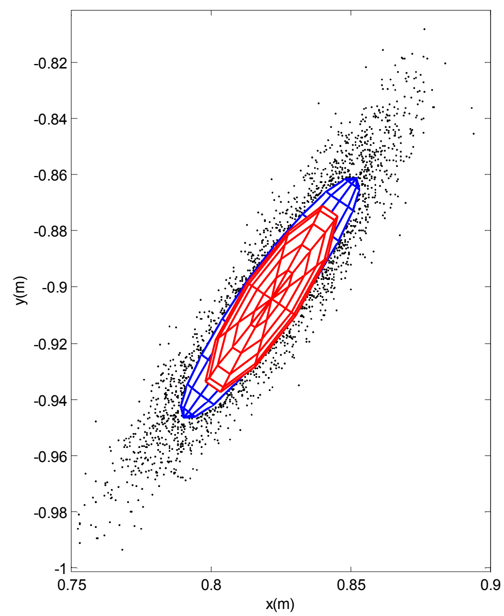

Figure 2. PDFs obtained via Monte-Carlo (blue) and via the linearization method presented in the paper (red).

ACKNOWLEGMENTS

This work is supported by FAPEMIG (Fundação de Amparo à Pesquisa do Estado de Minas Gerais). G. A. S. Pereira holds a scholarship from CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico).

REFERENCES

[1] J. Weston and D. Titterton, "Modern inertial navigation technology and its application," *Electronics & communication engineering journal*, vol. 12, no. 2, pp. 49-64, 2000.

[2] J. Xiaoxiong, L. Yu, S Baoku, and L. Ming, Experimental investigation on calibrating high precision

accelerometer on two-axis table," *Space Control and Inertial Technology Research Center, Harbin Institute of Technology, Harbin, Heilongjiang, China*, 2008.

[3] E. L. Renk, W. Collins, M. Rizzo, F. Lee, and D. S. Bernstein, "Calibrating a triaxial accelerometer-magnetometer," *IEEE Control Systems Magazine*, pp. 86-95, Dec. 2005.

[4] L. E. R. Silva, G. A. S. Pereira, and L. A. B. Torres, "Calibração de acelerômetros utilizando um robô manipulador industrial", Simpósio Brasileiro de Automação Inteligente, 2011.

[5] I. Lira, *Evaluating the measurement uncertainty: Fundamentals and practical guidance*. Institute of Physics Publishing, 2002

[6] J. J. Craig, *Introduction to Robotics: Mechanics and Control*. Pearson Prentice Hall, 2005.

[7] M. A. F. Martins, R. A. Kalid, G. A. Nery, L. A. Teixeira, and G. A. A. Gonçalves, "Comparação entre os métodos linear e não linear para a avaliação da incerteza de medição," *Controle & Automação*, vol 21, pp. 557 - 576, 2010.

[8] P. I. Corke, "MATLAB toolboxes: robotics and vision for students and teachers", IEEE Robotics and Automation Magazine, Volume 14(4), December 2007, pp. 16-17

[9] JCGM 100:2008. "Evaluation of measurement data – Guide to the expression of uncertainty in measurement", found at the internet at http://www.bipm.org/en/publications/guides/gum.html in 01/07/2011.

[10] JCGM 200:2008,. International Vocabulary of Metrology – Basic and General Concepts and Associated Terms, VIM, 3rd edition, found at the internet at http://www.bipm.org/utils/common/documents/jcgm/JCGM_200_2008.pdf. in 01/07/2011.